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A BINOMIÁLIS EGYÜTTHATÓK TULAJDONSÁGAI!

A megismerés térben és időben végtelen.
Vedd ki a számot a dolgokból, és minden összeomlik.
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I. BEVEZETÉS

A binomiális együtthatók önálló objektumok a matematikában, ezért a binomiális tételtől függetlenül sok területen használatosak (kombinatorika, valószínűség számítás, műszaki tudományok, közgazdaságtan, természettudományok –kémia, fizika, biológia-,…). A binomiális együtthatók vizsgálata, több tulajdonságukat hozott a felszínre, most csak két tulajdonsággal kívánok foglalkozni, pontosabban, az $\binom{n}{0}-\binom{n}{1}+\binom{n}{2}-\ldots\pm\binom{n}{n}=0,\ n>0$ kiterjesztésével, illetve a bizonyításoknál megemlítem még a szimmetria tulajdonságot. A káoszelméletben használatos ts(t-s) logisztikus egyenlet magasabb fokú változatainak differenciálásával adódnak ezek az új tulajdonságok.

Tétel 0

$$[m_{1,1}+nm_{2,1}][m_{1,2}+nm_{1,2}][m_{1,3}+nm_{2,3}]\ldots[m_{1,i}+nm_{2,i}]l_{1,1}l_{1,2}l_{1,3}\ldots l_{1,3}\binom{n}{0}-$$

$$-[m_{1,1}+(n-1)m_{2,1}][m_{1,2}+(n-1)m_{1,2}][m_{1,3}+(n-1)m_{2,3}]\ldots[m_{1,i}+(n-1)m_{2,i}]\cdot$$

$$\cdot[l_{1,1}+l_{2,1}][l_{1,2}+l_{2,2}][l_{1,3}+l_{2,3}]\ldots[l_{1,j}+l_{2,j}]\binom{n}{1}+$$

$$+[m_{1,1}+(n-2)m_{2,1}][m_{1,2}+(n-2)m_{1,2}][m_{1,3}+(n-2)m_{2,3}]\ldots[m_{1,i}+(n-2)m_{2,i}]\cdot$$

$$\cdot[l_{1,1}+2l_{2,1}][l_{1,2}+2l_{2,2}][l_{1,3}+2l_{2,3}]\ldots[l_{1,j}+2l_{2,j}]\binom{n}{2}-\ldots\pm$$

$$\pm m_{1,1}m_{1,2}m_{1,3}\ldots m_{1,i}[l_{1,1}+nl_{2,1}][l_{1,2}+nl_{2,2}][l_{1,3}+nl_{2,3}]\ldots[l_{1,j}+nl_{2,j}]\binom{n}{n}=$$

$$
=\begin{cases}
0,\ ha\ i+j<n \\
-n!\,m_{2,1}m_{2,2}m_{2,2}...m_{2,i}l_{2,1}l_{2,2}l_{2,2}...l_{2,j},\ ha\ i+j=n,\ j=2k-1,\ k=1,2,3,... \\
n!\,m_{2,1}m_{2,2}m_{2,2}...m_{2,i}l_{2,1}l_{2,2}l_{2,2}...l_{2,j},\ ha\ i+j=n,\ j=2k,\ k=1,2,3,... \\
<0,\ ha\ n<i+j<2n,\ n=2k,\ k=1,2,3,... \\
<0,\ ha\ n<i+j<2n,\ n=2k-1,\ k=1,2,3,... \\
>0,\ ha\ n<i+j<2n,\ n=2k-1,\ j=2k-1,\ k=1,2,3,... \\
0,\ ha\ i+j=2n,\ n=1,3,5,... \\
<0,\ ha\ i+j=2n,\ n=2k-1,\ k=1,2,3,... \\
>0,\ ha\ i+j=2n,\ n=2k,\ k=1,2,3,...
\end{cases}
$$

Bizonyítása a

$$
t_1^{m_{1,1}}t_2^{m_{1,2}}t_3^{m_{1,3}}\cdots t_n^{m_{1,n}}s_1^{l_{1,1}}s_2^{l_{1,2}}s_3^{l_{1,3}}\cdots s_n^{l_{1,n}}(t_1^{m_{2,1}}t_2^{m_{2,2}}t_3^{m_{2,3}}\cdots t_n^{m_{2,n}}-s_1^{l_{2,1}}s_2^{l_{2,2}}s_3^{l_{2,3}}\cdots s_n^{l_{2,n}})^n=
$$

$$
=t_1^{m_{1,1}+nm_{2,1}}t_2^{m_{1,2}+nm_{2,2}}t_3^{m_{1,3}+nm_{2,3}}\cdots t_n^{m_{1,n}+nm_{2,n}}s_1^{l_{1,1}}s_2^{l_{1,2}}s_3^{l_{1,3}}\cdots s_n^{l_{1,n}}\binom{n}{0}-
$$

$$
-t_1^{m_{1,1}+(n-1)m_{2,1}}t_2^{m_{1,2}+(n-1)m_{2,2}}t_3^{m_{1,3}+(n-1)m_{2,3}}\cdots t_n^{m_{1,n}+(n-1)m_{2,n}}s_1^{l_{1,1}+l_{2,1}}s_2^{l_{1,2}+l_{2,2}}s_3^{l_{1,3}+l_{2,3}}\cdots s_n^{l_{1,n}+l_{2,n}}\binom{n}{1}+
$$

$$
+t_1^{m_{1,1}+(n-2)m_{2,1}}t_2^{m_{1,2}+(n-2)m_{2,2}}t_3^{m_{1,3}+(n-2)m_{2,3}}\cdots t_n^{m_{1,n}+(n-2)m_{2,n}}s_1^{l_{1,1}+2l_{2,1}}s_2^{l_{1,2}+2l_{2,2}}s_3^{l_{1,3}+2l_{2,3}}\cdots s_n^{l_{1,n}+2l_{2,n}}\binom{n}{2}-
$$

$$
-....\pm t_1^{m_{1,1}}t_2^{m_{1,2}}t_3^{m_{1,3}}\cdots t_n^{m_{1,n}}s_1^{l_{1,1}+nl_{2,1}}s_2^{l_{1,2}+nl_{2,2}}s_3^{l_{1,3}+nl_{2,3}}\cdots s_n^{l_{1,n}+nl_{2,n}}\binom{n}{n}
$$

azonosságot deriváljuk t_i, és s_j, i,j=1,2,3,…,n változók szerint, és minden deriválás után a t_i, és s_j, i,j=1,2,3,…,n változók értékét egyenlővé tesszük 1-el, adódik az állítás. A baloldalon a deriválások után minden tag tartalmaz $(t_1^{m_{2,1}}t_2^{m_{2,2}}t_3^{m_{2,3}}\cdots t_n^{m_{2,n}}-s_1^{l_{2,1}}s_2^{l_{2,2}}s_3^{l_{2,3}}\cdots s_n^{l_{2,n}})^k$, k=1,2,3,…,n, addig, míg az i+j el nem éri az n-et, így a bal oldal 0, következésképp a jobb oldal is 0. Amennyiben a deriválások száma legalább n, és legfeljebb 2n, a bal oldalon szerepelnek olyan tagok, amelyek nem tartalmaznak $(t_1^{m_{2,1}}t_2^{m_{2,2}}t_3^{m_{2,3}}\cdots t_n^{m_{2,n}}-s_1^{l_{2,1}}s_2^{l_{2,2}}s_3^{l_{2,3}}\cdots s_n^{l_{2,n}})$ tényezőt, ebből az is következik, hogy a bal oldal nem egyenlő 0-val, következésképpen a jobb oldal sem egyenlő 0-val.

A bizonyításoknál a szimmetria tulajdonságot is felhasználhatjuk!
A **Tétel 0** tartalmazza speciális esetként az összes állítást!
n=1

$$
t_1^{m_{1,1}}s_1^{l_{1,1}}(t_1^{m_{2,1}}-s_1^{l_{2,1}})=t_1^{m_{1,1}+1m_{2,1}}s_1^{l_{1,1}}\binom{1}{0}-t_1^{m_{1,1}}s_1^{l_{1,1}+1l_{2,1}}\binom{1}{1}
$$

t_1-szerint deriválunk!

$$
m_{1,1}t_1^{m_{1,1}-1}s_1^{l_{1,1}}(t_1^{m_{2,1}}-s_1^{l_{2,1}})+m_{2,1}t_1^{m_{1,1}}s_1^{l_{1,1}}t_1^{m_{2,1}-1}=[m_{1,1}+m_{2,1}]t_1^{m_{1,1}+1m_{2,1}-1}s_1^{l_{1,1}}\binom{1}{0}-m_{1,1}t_1^{m_{1,1}-1}s_1^{l_{1,1}+1l_{2,1}}\binom{1}{1}
$$

$t_1=s_1=1$
$m_{2,1}=[m_{1,1}+m_{2,1}]-m_{1,1}=m_{2,1}$
$m_{1,1}=m_{2,1}=1$, esetén 1!

s_1-szerint deriválunk!

$$
l_{1,1}t_1^{m_{1,1}}s_1^{l_{1,1}-1}(t_1^{m_{2,1}}-s_1^{l_{2,1}})-l_{2,1}t_1^{m_{1,1}}s_1^{l_{1,1}}s_1^{l_{2,1}-1}=l_{1,1}t_1^{m_{1,1}+1m_{2,1}}s_1^{l_{1,1}-1}\binom{1}{0}-[l_{1,1}+l_{2,1}]t_1^{m_{1,1}}s_1^{l_{1,1}+1l_{2,1}-1}\binom{1}{1}
$$

$t_1=s_1=1$
$-l_{2,1}=l_{1,1}-[l_{1,1}+l_{2,1}]$
$l_{1,1}=l_{2,1}=1$, esetén -1!

Előbb t_1, majd s_1-szerint deriválunk!

$$m_{1,1}l_{1,1}t_1^{m_{1,1}-1}s_1^{l_{1,1}-1}(t_1^{m_{2,1}}-s_1^{l_{2,1}})-m_{1,1}l_{1,2}t_1^{m_{1,1}-1}s_1^{l_{1,1}}s_1^{l_{2,1}-1}+m_{2,1}l_{1,1}t_1^{m_{1,1}}t_1^{m_{2,1}-1}s_1^{l_{1,1}-1}=$$

$$=[m_{1,1}+m_{2,1}]l_{1,1}t_1^{m_{1,1}+1m_{2,1}-1}s_1^{l_{1,1}-1}\begin{pmatrix}1\\0\end{pmatrix}-m_{1,1}[l_{1,1}+l_{2,1}]t_1^{m_{1,1}-1}s_1^{l_{1,1}+1l_{2,1}-1}\begin{pmatrix}1\\1\end{pmatrix}$$

$t_1=s_1=1$
$-m_{1,1}l_{1,2}+m_{2,1}l_{1,1}=[m_{1,1}+m_{2,1}]l_{1,1}-m_{1,1}[l_{1,1}+l_{2,1}]=m_{1,1}l_{1,1}+m_{2,1}l_{1,1}-m_{1,1}l_{1,1}-m_{1,1}l_{2,1}=m_{2,1}l_{1,1}-m_{1,1}l_{2,1}$

$$[m_{1,1}+1m_{2,1}]l_{1,1}\begin{pmatrix}1\\0\end{pmatrix}-m_{1,1}[l_{1,1}+1l_{2,1}]\begin{pmatrix}1\\1\end{pmatrix}=[m_{1,1}+m_{2,1}]l_{1,1}-m_{1,1}[l_{1,1}+l_{2,1}]=m_{2,1}l_{1,1}-m_{1,1}l_{2,1}$$

m=l

$$[m_{1,1}+1m_{2,1}]m_{1,1}\begin{pmatrix}1\\0\end{pmatrix}-m_{1,1}[m_{1,1}+1m_{2,1}]\begin{pmatrix}1\\1\end{pmatrix}=[m_{1,1}+m_{2,1}]m_{1,1}-m_{1,1}[m_{1,1}+m_{2,1}]=m_{2,1}m_{1,1}-m_{1,1}m_{2,1}=0$$

n=2

$$t_1^{m_{1,1}}t_2^{m_{1,2}}s_1^{l_{1,1}}s_2^{l_{1,2}}(t_1^{m_{2,1}}t_2^{m_{2,2}}-s_1^{l_{2,1}}s_2^{l_{2,2}})^2=$$

$$=t_1^{m_{1,1}+2m_{2,1}}t_2^{m_{1,2}+2m_{2,2}}s_1^{l_{1,1}}s_2^{l_{1,2}}\begin{pmatrix}2\\0\end{pmatrix}-t_1^{m_{1,1}+m_{2,1}}t_2^{m_{1,2}+m_{2,2}}s_1^{l_{1,1}+l_{2,1}}s_2^{l_{1,2}+l_{2,2}}\begin{pmatrix}2\\1\end{pmatrix}+t_1^{m_{1,1}}t_2^{m_{1,2}}s_1^{l_{1,1}+2l_{2,1}}s_2^{l_{1,2}+2l_{2,2}}\begin{pmatrix}2\\2\end{pmatrix}$$

t_1-szerint deriválunk!

$$m_{1,1}t_1^{m_{1,1}-1}t_2^{m_{1,2}}s_1^{l_{1,1}}s_2^{l_{1,2}}(t_1^{m_{2,1}}t_2^{m_{2,2}}-s_1^{l_{2,1}}s_2^{l_{2,2}})^2+2m_{2.1}t_1^{m_{1,1}}t_2^{m_{1,2}}s_1^{l_{1,1}}s_2^{l_{1,2}}(t_1^{m_{2,1}}t_2^{m_{2,2}}-s_1^{l_{2,1}}s_2^{l_{2,2}})t_1^{m_{2,1}-1}=$$

$$=[m_{1,1}+2m_{2.1}]t_1^{m_{1,1}+2m_{2,1}-1}t_2^{m_{1,2}+2m_{2,2}}s_1^{l_{1,1}}s_2^{l_{1,2}}\begin{pmatrix}2\\0\end{pmatrix}-$$

$$-[m_{1,1}+m_{2,1}]t_1^{m_{1,1}+m_{2,1}-1}t_2^{m_{1,2}+m_{2,2}}s_1^{l_{1,1}+l_{2,1}}s_2^{l_{1,2}+l_{2,2}}\begin{pmatrix}2\\1\end{pmatrix}+m_{1,1}t_1^{m_{1,1}-1}t_2^{m_{1,2}}s_1^{l_{1,1}+2l_{2,1}}s_2^{l_{1,2}+2l_{2,2}}\begin{pmatrix}2\\2\end{pmatrix}$$

$t_1=t_2=s_1=s_2=1$

$$0=[m_{1,1}+2m_{2.1}]\begin{pmatrix}2\\0\end{pmatrix}-[m_{1,1}+m_{2.1}]\begin{pmatrix}2\\1\end{pmatrix}+m_{1,1}\begin{pmatrix}2\\2\end{pmatrix}=[m_{1,1}+2m_{2.1}]-[m_{1,1}+m_{2.1}]2+m_{1,1}=0$$

t_2-szerint deriválunk!

$$m_{1,2}t_1^{m_{1,1}}t_2^{m_{1,2}-1}s_1^{l_{1,1}}s_2^{l_{1,2}}(t_1^{m_{2,1}}t_2^{m_{2,2}}-s_1^{l_{2,1}}s_2^{l_{2,2}})^2+m_{2,2}t_1^{m_{1,1}}t_2^{m_{1,2}-1}s_1^{l_{1,1}}s_2^{l_{1,2}}(t_1^{m_{2,1}}t_2^{m_{2,2}}-s_1^{l_{2,1}}s_2^{l_{2,2}})t_2^{m_{2,2}-1}=$$

$$=[m_{1,2}+2m_{2,2}]t_1^{m_{1,1}+2m_{2,1}}t_2^{m_{1,2}+2m_{2,2}-1}s_1^{l_{1,1}}s_2^{l_{1,2}}\begin{pmatrix}2\\0\end{pmatrix}-$$

$$-[m_{1,2}+m_{2,2}]t_1^{m_{1,1}+m_{2,1}}t_2^{m_{1,2}+m_{2,2}-1}s_1^{l_{1,1}+l_{2,1}}s_2^{l_{1,2}+l_{2,2}}\begin{pmatrix}2\\1\end{pmatrix}+m_{1,2}t_1^{m_{1,1}}t_2^{m_{1,2}-1}s_1^{l_{1,1}+2l_{2,1}}s_2^{l_{1,2}+2l_{2,2}}\begin{pmatrix}2\\2\end{pmatrix}$$

$t_1=t_2=s_1=s_2=1$

$$0=[m_{1,2}+2m_{2,2}]\begin{pmatrix}2\\0\end{pmatrix}-[m_{1,2}+m_{2,2}]\begin{pmatrix}2\\1\end{pmatrix}+m_{1,2}\begin{pmatrix}2\\2\end{pmatrix}=[m_{1,2}+2m_{2,2}]-[m_{1,2}+m_{2,2}]2+m_{1,2}=0$$

Előbb t_1, majd t_2-szerint deriválunk!

$$m_{1,1}m_{1,2}t_1^{m_{1,1}-1}t_2^{m_{1,2}-1}s_1^{l_{1,1}}s_2^{l_{1,2}}(t_1^{m_{2,1}}t_2^{m_{2,2}}-s_1^{l_{2,1}}s_2^{l_{2,2}})^2+2m_{1,1}m_{2,2}t_1^{m_{1,1}-1}t_2^{m_{1,2}}s_1^{l_{1,1}}s_2^{l_{1,2}}(t_1^{m_{2,1}}t_2^{m_{2,2}}-s_1^{l_{2,1}}s_2^{l_{2,2}})t_2^{m_{2,2}-1}+$$

$$+2m_{2.1}m_{1,2}t_1^{m_{1,1}}t_2^{m_{1,2}-1}s_1^{l_{1,1}}s_2^{l_{1,2}}(t_1^{m_{2,1}}t_2^{m_{2,2}}-s_1^{l_{2,1}}s_2^{l_{2,2}})t_1^{m_{2,1}-1}+2\cdot 1m_{2,1}m_{2,2}t_1^{m_{1,1}}t_2^{m_{1,2}}s_1^{l_{1,1}}s_2^{l_{1,2}}t_1^{m_{2,1}-1}t_2^{m_{2,2}-1}=$$

$$=[m_{1,1}+2m_{2,.1}][m_{1,2}+2m_{2,.2}]t_1^{m_{1,1}+2m_{2,1}-1}t_2^{m_{1,2}+2m_{2,2}-1}s_1^{l_{1,1}}s_2^{l_{1,2}}\binom{2}{0}-$$

$$-[m_{1,1}+m_{2.1}][m_{1,2}+m_{2,.2}]t_1^{m_{1,1}+m_{2,1}-1}t_2^{m_{1,2}+m_{2,2}-1}s_1^{l_{1,1}+l_{2,1}}s_2^{l_{1,2}+l_{2,2}}\binom{2}{1}+m_{1,1}m_{1,2}t_1^{m_{1,1}-1}t_2^{m_{1,2}-1}s_1^{l_{1,1}+2l_{2,1}}s_2^{l_{1,2}+2l_{2,2}}\binom{2}{2}$$

$t_1=t_2=s_1=s_2=1$

$$2\cdot 1m_{2,1}m_{2,2}=2!m_{2,1}m_{2,2}=[m_{1,1}+2m_{2,1}][m_{1,2}+2m_{2,.2}]\binom{2}{0}-[m_{1,1}+m_{2.1}][m_{1,2}+m_{2,.2}]\binom{2}{1}+m_{1,1}m_{1,2}\binom{2}{2}=$$

$$=[m_{1,1}+2m_{2,1}][m_{1,2}+2m_{2,.2}]-[m_{1,1}+m_{2.1}][m_{1,2}+m_{2,.2}]2+m_{1,1}m_{1,2}=2m_{2,1}m_{2,2}$$

$m_{1,1}=m_{1,2}=m_{2,1}=m_{2,1}=1$, esetén 2!

s_1-szerint deriválunk!

$$l_{1,1}t_1^{m_{1,1}}t_2^{m_{1,2}}s_1^{l_{1,1}-1}s_2^{l_{1,2}}(t_1^{m_{2,1}}t_2^{m_{2,2}}-s_1^{l_{2,1}}s_2^{l_{2,2}})^2-2l_{2,1}t_1^{m_{1,1}}t_2^{m_{1,2}}s_1^{l_{1,1}}s_2^{l_{1,2}}(t_1^{m_{2,1}}t_2^{m_{2,2}}-s_1^{l_{2,1}}s_2^{l_{2,2}})s_1^{l_{2,1}-1}=$$

$$=l_{1,1}t_1^{m_{1,1}+2m_{2,1}}t_2^{m_{1,2}+2m_{2,2}}s_1^{l_{1,1}-1}s_2^{l_{1,2}}\binom{2}{0}-[l_{1,1}+l_{2,1}]t_1^{m_{1,1}+m_{2,1}}t_2^{m_{1,2}+m_{2,2}}s_1^{l_{1,1}+l_{2,1}-1}s_2^{l_{1,2}+l_{2,2}}\binom{2}{1}+$$

$$+[l_{1,1}+2l_{2,1}]t_1^{m_{1,1}}t_2^{m_{1,2}}s_1^{l_{1,1}+2l_{2,1}-1}s_2^{l_{1,2}+2l_{2,2}}\binom{2}{2}$$

$t_1=t_2=s_1=s_2=1$

$$0=l_{1,1}\binom{2}{0}-[l_{1,1}+l_{2,1}]\binom{2}{1}+[l_{1,1}+2l_{2,1}]\binom{2}{2}=l_{1,1}-[l_{1,1}+l_{2,1}]2+[l_{1,1}+2l_{2,1}]=0$$

s_2-szerint deriválunk!

$$l_{1,2}t_1^{m_{1,1}}t_2^{m_{1,2}}s_1^{l_{1,1}}s_2^{l_{1,2}-1}(t_1^{m_{2,1}}t_2^{m_{2,2}}-s_1^{l_{2,1}}s_2^{l_{2,2}})^2-2l_{2,2}t_1^{m_{1,1}}t_2^{m_{1,2}}s_1^{l_{1,1}}s_2^{l_{1,2}-1}(t_1^{m_{2,1}}t_2^{m_{2,2}}-s_1^{l_{2,1}}s_2^{l_{2,2}})=$$

$$=l_{1,2}t_1^{m_{1,1}+2m_{2,1}}t_2^{m_{1,2}+2m_{2,2}}s_1^{l_{1,1}}s_2^{l_{1,2}-1}\binom{2}{0}-[l_{1,2}+l_{2,2}]t_1^{m_{1,1}+m_{2,1}}t_2^{m_{1,2}+m_{2,2}}s_1^{l_{1,1}+l_{2,1}}s_2^{l_{1,2}+l_{2,2}-1}\binom{2}{1}+$$

$$+[l_{1,2}+2l_{2,2}]t_1^{m_{1,1}}t_2^{m_{1,2}}s_1^{l_{1,1}+2l_{2,1}}s_2^{l_{1,2}+2l_{2,2}-1}\binom{2}{2}$$

$t_1=t_2=s_1=s_2=1$

$$0=l_{1,2}\binom{2}{0}-[l_{1,2}+l_{2,2}]\binom{2}{1}+[l_{1,2}+2l_{2,2}]\binom{2}{2}=l_{1,2}-[l_{1,2}+l_{2,2}]2+[l_{1,2}+2l_{2,2}]=0$$

Előbb s_1, majd s_2-szerint deriválunk!

$$l_{1,1}l_{1,2}t_1^{m_{1,1}}t_2^{m_{1,2}}s_1^{l_{1,1}-1}s_2^{l_{1,2}-1}(t_1^{m_{2,1}}t_2^{m_{2,2}}-s_1^{l_{2,1}}s_2^{l_{2,2}})^2-2l_{1,1}l_{2,2}t_1^{m_{1,1}}t_2^{m_{1,2}}s_1^{l_{1,1}-1}s_2^{l_{1,2}}(t_1^{m_{2,1}}t_2^{m_{2,2}}-s_1^{l_{2,1}}s_2^{l_{2,2}})s_2^{l_{2,2}-1}$$

$$-2l_{2,1}t_1^{m_{1,1}}t_2^{m_{1,2}}s_1^{l_{1,1}}s_2^{l_{1,2}}(t_1^{m_{2,1}}t_2^{m_{2,2}}-s_1^{l_{2,1}}s_2^{l_{2,2}})s_1^{l_{2,1}-1}-2l_{1,2}l_{2,1}t_1^{m_{1,1}}t_2^{m_{1,2}}s_1^{l_{1,1}}s_2^{l_{1,2}-1}(t_1^{m_{2,1}}t_2^{m_{2,2}}-s_1^{l_{2,1}}s_2^{l_{2,2}})s_1^{l_{2,1}-1}+$$

$$+2\cdot 1l_{2,1}l_{2,2}t_1^{m_{1,1}}t_2^{m_{1,2}}s_1^{l_{1,1}}s_2^{l_{1,2}}s_1^{l_{2,1}-1}s_2^{l_{2,2}-1}=$$

$$=l_{1,1}l_{1,2}t_1^{m_{1,1}+2m_{2,1}}t_2^{m_{1,2}+2m_{2,2}}s_1^{l_{1,1}-1}s_2^{l_{1,2}-1}\binom{2}{0}-[l_{1,1}+l_{2,1}][l_{1,2}+l_{2,2}]t_1^{m_{1,1}+m_{2,1}}t_2^{m_{1,2}+m_{2,2}}s_1^{l_{1,1}+l_{2,1}-1}s_2^{l_{1,2}+l_{2,2}-1}\binom{2}{1}+$$

$$+[l_{1,1}+2l_{2,1}][l_{1,2}+2l_{2,2}]t_1^{m_{1,1}}t_2^{m_{1,2}}s_1^{l_{1,1}+2l_{2,1}-1}s_2^{l_{1,2}+2l_{2,2}-1}\binom{2}{2}$$

$t_1=t_2=s_1=s_2=1$

$$2\cdot 1 l_{2,1}l_{2,2}=l_{1,1}l_{1,2}\binom{2}{0}-[l_{1,1}+l_{2,1}][l_{1,2}+l_{2,2}]\binom{2}{1}+[l_{1,1}+2l_{2,1}][l_{1,2}+2l_{2,2}]\binom{2}{2}=$$

$$=l_{1,1}l_{1,2}-[l_{1,1}+l_{2,1}][l_{1,2}+l_{2,2}]2+[l_{1,1}+2l_{2,1}][l_{1,2}+2l_{2,2}]=2l_{2,1}l_{2,2}$$

$l_{1,1}=l_{1,2}=l_{2,1}=l_{2,1}=1$, esetén 2!

Deriváljunk előbb t_1,t_2, majd s_1 szerint!

$$[m_{1,1}+2m_{2,1}][m_{1,2}+2m_{2,.2}]l_{1,1}t_1^{m_{1,1}+2m_{2,1}-1}t_2^{m_{1,2}+2m_{2,2}-1}s_1^{l_{1,1}-1}s_2^{l_{1,2}}\binom{2}{0}-$$

$$-[m_{1,1}+m_{2.1}][m_{1,2}+m_{2,.2}][l_{1,1}+l_{2,1}]t_1^{m_{1,1}+m_{2,1}-1}t_2^{m_{1,2}+m_{2,2}-1}s_1^{l_{1,1}+l_{2,1}-1}s_2^{l_{1,2}+l_{2,2}}\binom{2}{1}+$$

$$+m_{1,1}m_{1,2}[l_{1,1}+2l_{2,1}]t_1^{m_{1,1}-1}t_2^{m_{1,2}-1}s_1^{l_{1,1}+2l_{2,1}-1}s_2^{l_{1,2}+2l_{2,2}}\binom{2}{2}$$

$t_1=t_2=s_1=s_2=1$

$$[m_{1,1}+2m_{2,1}][m_{1,2}+2m_{2,.2}]l_{1,1}\binom{2}{0}-[m_{1,1}+m_{2,1}][m_{1,2}+m_{2,.2}][l_{1,1}+l_{2.1}]\binom{2}{1}+m_{1,1}m_{1,2}[l_{1,1}+2l_{2.1}]\binom{2}{2}=$$

$$=[m_{1,1}+2m_{2,1}][m_{1,2}+2m_{2,.2}]l_{1,1}-[m_{1,1}+m_{2.1}][m_{1,2}+m_{2,.2}][l_{1,1}+l_{2.1}]2+m_{1,1}m_{1,2}[l_{1,1}+2l_{2,1}]$$

$m_{1,1}=m_{1,2}=m_{2,1}=m_{2,1}=l_{1,1}=l_{1,2}=l_{2,1}=l_{2,2}=1$, esetén -4

Deriváljunk előbb t_1,t_2, majd s_1,s_2, szerint!

$$[m_{1,1}+2m_{2,1}][m_{1,2}+2m_{2,.2}]l_{1,1}l_{1,2}t_1^{m_{1,1}+2m_{2,1}-1}t_2^{m_{1,2}+2m_{2,2}-1}s_1^{l_{1,1}-1}s_2^{l_{1,2}-1}\binom{2}{0}-$$

$$-[m_{1,1}+m_{2.1}][m_{1,2}+m_{2,.2}][l_{1,1}+l_{2,1}][l_{1,2}+l_{2,.2}]t_1^{m_{1,1}+m_{2,1}-1}t_2^{m_{1,2}+m_{2,2}-1}s_1^{l_{1,1}+l_{2,1}-1}s_2^{l_{1,2}+l_{2,2}}\binom{2}{1}+$$

$$+m_{1,1}m_{1,2}[l_{1,1}+2l_{2,1}][l_{1,2}+2l_{2,2}]t_1^{m_{1,1}-1}t_2^{m_{1,2}-1}s_1^{l_{1,1}+2l_{2,1}-1}s_2^{l_{1,2}+2l_{2,2}}\binom{2}{2}$$

$m_{1,1}=m_{1,2}=m_{2,1}=m_{2,1}=l_{1,1}=l_{1,2}=l_{2,1}=l_{2,2}=1$, esetén -14

$n=3$

$$t_1^{m_{1,1}}t_2^{m_{1,2}}t_3^{m_{1,3}}s_1^{l_{1,1}}s_2^{l_{1,2}}s_3^{l_{1,3}}(t_1^{m_{2,1}}t_2^{m_{2,2}}t_3^{m_{2,3}}-s_1^{l_{2,1}}s_2^{l_{2,2}}s_3^{l_{2,3}})^3=$$

$$=t_1^{m_{1,1}+3m_{2,1}}t_2^{m_{1,2}+3m_{2,2}}t_3^{m_{1,3}+3m_{2,3}}s_1^{l_{1,1}}s_2^{l_{1,2}}s_3^{l_{1,3}}\binom{3}{0}-t_1^{m_{1,1}+2m_{2,1}}t_2^{m_{1,2}+2m_{2,2}}t_3^{m_{1,3}+2m_{2,3}}s_1^{l_{1,1}+l_{2,1}}s_2^{l_{1,2}+l_{2,2}}s_3^{l_{1,2}+l_{2,3}}\binom{3}{1}+$$

$$+t_1^{m_{1,1}+m_{2,1}}t_2^{m_{1,2}+m_{2,2}}t_3^{m_{1,3}+m_{2,3}}s_1^{l_{1,1}+2l_{2,1}}s_2^{l_{1,2}+2l_{2,2}}s_3^{l_{1,2}+2l_{2,3}}\binom{3}{2}-t_1^{m_{1,1}}t_2^{m_{1,2}}t_3^{m_{1,3}}s_1^{l_{1,1}+3l_{2,1}}s_2^{l_{1,2}+3l_{2,2}}s_3^{l_{1,2}+3l_{2,3}}\binom{3}{3}$$

t_1-szerint deriválunk!

$$[m_{1,1}+3m_{2,1}]t_1^{m_{1,1}+3m_{2,1}-1}t_2^{m_{1,2}+3m_{2,2}}t_3^{m_{1,3}+3m_{2,3}}s_1^{l_{1,1}}s_2^{l_{1,2}}s_3^{l_{1,3}}\binom{3}{0}-$$

$$-[m_{1,1}+2m_{2,1}]t_1^{m_{1,1}+2m_{2,1}-1}t_2^{m_{1,2}+2m_{2,2}}t_3^{m_{1,3}+2m_{2,3}}s_1^{l_{1,1}+l_{2,1}}s_2^{l_{1,2}+l_{2,2}}s_3^{l_{1,3}+l_{2,3}}\binom{3}{1}+$$

$$+[m_{1,1}+m_{2,1}]t_1^{m_{1,1}+m_{2,1}-1}t_2^{m_{1,2}+m_{2,2}}t_3^{m_{1,3}+m_{2,3}}s_1^{l_{1,1}+2l_{2,1}}s_2^{l_{1,2}+2l_{2,2}}s_3^{l_{1,3}+2l_{2,3}}\binom{3}{2}-$$

$$-m_{1,1}t_1^{m_{1,1}-1}t_2^{m_{1,2}}t_3^{m_{1,3}}s_1^{l_{1,1}+3l_{2,1}}s_2^{l_{1,2}+3l_{2,2}}s_3^{l_{1,3}+3l_{2,3}}\binom{3}{3}$$

$t_1=t_2=t_3=s_1=s_2=s_3=1$

$$[m_{1,1}+3m_{2,1}]\binom{3}{0}-[m_{1,1}+2m_{2,1}]\binom{3}{1}+[m_{1,1}+m_{2,1}]\binom{3}{2}-m_{1,1}\binom{3}{3}=$$

$$=[m_{1,1}+3m_{2,1}]-[m_{1,1}+2m_{2,1}]3+[m_{1,1}+m_{2,1}]3-m_{1,1}=0$$

t_2-szerint deriválunk!

$$[m_{1,2}+3m_{2,2}]t_1^{m_{1,1}+3m_{2,1}}t_2^{m_{1,2}+3m_{2,2}-1}t_3^{m_{1,2}+3m_{2,3}}s_1^{l_{1,1}}s_2^{l_{1,2}}s_3^{l_{1,3}}\binom{3}{0}-$$

$$-[m_{1,2}+2m_{2,2}]t_1^{m_{1,1}+2m_{2,1}}t_2^{m_{1,2}+2m_{2,2}-1}t_3^{m_{1,3}+2m_{2,3}}s_1^{l_{1,1}+l_{2,1}}s_2^{l_{1,2}+l_{2,2}}s_3^{l_{1,3}+l_{2,3}}\binom{3}{1}+$$

$$+[m_{1,2}+m_{2,2}]t_1^{m_{1,1}+m_{2,1}}t_2^{m_{1,2}+m_{2,2}-1}t_3^{m_{1,3}+m_{2,3}}s_1^{l_{1,1}+2l_{2,1}}s_2^{l_{1,2}+2l_{2,2}}s_3^{l_{1,2}+2l_{2,3}}\binom{3}{2}-$$

$$-m_{1,2}t_1^{m_{1,1}}t_2^{m_{1,2}-1}t_3^{m_{1,3}}s_1^{l_{1,1}+3l_{2,1}}s_2^{l_{1,2}+3l_{2,2}}s_3^{l_{1,2}+3l_{2,3}}\binom{3}{3}$$

$t_1=t_2=t_3=s_1=s_2=s_3=1$

$$[m_{1,2}+3m_{2,2}]\binom{3}{0}-[m_{1,2}+2m_{2,2}]\binom{3}{1}+[m_{1,2}+m_{2,2}]\binom{3}{2}-m_{1,2}\binom{3}{3}=$$

$$=[m_{1,2}+3m_{2,2}]-[m_{1,2}+2m_{2,2}]3+[m_{1,2}+m_{2,2}]3-m_{1,2}=0$$

t_2-szerint deriválunk!

$$[m_{1,2}+3m_{2,2}]t_1^{m_{1,1}+3m_{2,1}}t_2^{m_{1,2}+3m_{2,2}-1}t_3^{m_{1,3}+3m_{2,3}}s_1^{l_{1,1}}s_2^{l_{1,2}}s_3^{l_{1,3}}\binom{3}{0}-$$

$$-[m_{1,2}+2m_{2,2}]t_1^{m_{1,1}+2m_{2,1}}t_2^{m_{1,2}+2m_{2,2}-1}t_3^{m_{1,3}+2m_{2,3}}s_1^{l_{1,1}+l_{2,1}}s_2^{l_{1,2}+l_{2,2}}s_3^{l_{1,3}+l_{2,3}}\binom{3}{1}+$$

$$+[m_{1,2}+m_{2,2}]t_1^{m_{1,1}+m_{2,1}}t_2^{m_{1,2}+m_{2,2}-1}t_3^{m_{1,3}+m_{2,3}}s_1^{l_{1,1}+2l_{2,1}}s_2^{l_{1,2}+2l_{2,2}}s_3^{l_{1,3}+2l_{2,3}}\binom{3}{2}-$$

$$-m_{1,2}t_1^{m_{1,1}}t_2^{m_{1,2}-1}t_3^{m_{1,3}}s_1^{l_{1,1}+3l_{2,1}}s_2^{l_{1,2}+3l_{2,2}}s_3^{l_{1,3}+3l_{2,3}}\binom{3}{3}$$

$t_1=t_2=t_3=s_1=s_2=s_3=1$

$$[m_{1,2}+3m_{2,2}]\binom{3}{0}-[m_{1,2}+2m_{2,2}]\binom{3}{1}+[m_{1,2}+m_{2,2}]\binom{3}{2}-m_{1,2}\binom{3}{3}=$$

$$=[m_{1,2}+3m_{2,2}]-[m_{1,2}+2m_{2,2}]3+[m_{1,2}+m_{2,2}]3-m_{1,2}=0$$

t_3-szerint deriválunk!

$$[m_{1,3}+3m_{2,3}]t_1^{m_{1,1}+3m_{2,1}}t_2^{m_{1,2}+3m_{2,2}}t_3^{m_{1,3}+3m_{2,3}-1}s_1^{l_{1,1}}s_2^{l_{1,2}}s_3^{l_{1,3}}\binom{3}{0}-$$

$$-[m_{1,3}+2m_{2,3}]t_1^{m_{1,1}+2m_{2,1}}t_2^{m_{1,2}+2m_{2,2}}t_3^{m_{1,3}+2m_{2,3}-1}s_1^{l_{1,1}+l_{2,1}}s_2^{l_{1,2}+l_{2,2}}s_3^{l_{1,2}+l_{2,3}}\binom{3}{1}+$$

$$+[m_{1,3}+m_{2,3}]t_1^{m_{1,1}+m_{2,1}}t_2^{m_{1,2}+m_{2,2}}t_3^{m_{1,3}+m_{2,3}-1}s_1^{l_{1,1}+2l_{2,1}}s_2^{l_{1,2}+2l_{2,2}}s_3^{l_{1,2}+2l_{2,3}}\binom{3}{2}-$$

$$-m_{1,3}t_1^{m_{1,1}}t_2^{m_{1,2}}t_3^{m_{1,3}-1}s_1^{l_{1,1}+3l_{2,1}}s_2^{l_{1,2}+3l_{2,2}}s_3^{l_{1,2}+3l_{2,3}}\binom{3}{3}$$

$t_1=t_2=t_3=s_1=s_2=s_3=1$

$$[m_{1,3}+3m_{2,3}]\binom{3}{0}-[m_{1,3}+2m_{2,3}]\binom{3}{1}+[m_{1,3}+m_{2,3}]\binom{3}{2}-m_{1,3}\binom{3}{3}=$$

$$=[m_{1,3}+3m_{2,3}]-[m_{1,3}+2m_{2,3}]3+[m_{1,3}+m_{2,3}]3-m_{1,3}=0$$

Deriváljunk előbb t_1, majd t_2 szerint!

$$[m_{1,1}+3m_{2,1}][m_{1,2}+3m_{2,2}]t_1^{m_{1,1}+3m_{2,1}-1}t_2^{m_{1,2}+3m_{2,2}-1}t_3^{m_{1,3}+3m_{2,3}}s_1^{l_{1,1}}s_2^{l_{1,2}}s_3^{l_{1,3}}\binom{3}{0}-$$

$$-[m_{1,1}+2m_{2,1}][m_{1,2}+2m_{2,2}]t_1^{m_{1,1}+2m_{2,1}-1}t_2^{m_{1,2}+2m_{2,2}-1}t_3^{m_{1,3}+2m_{2,3}}s_1^{l_{1,1}+l_{2,1}}s_2^{l_{1,2}+l_{2,2}}s_3^{l_{1,3}+l_{2,3}}\binom{3}{1}+$$

$$+[m_{1,1}+m_{2,1}][m_{1,2}+m_{2,2}]t_1^{m_{1,1}+m_{2,1}-1}t_2^{m_{1,2}+m_{2,2}-1}t_3^{m_{1,3}+m_{2,3}}s_1^{l_{1,1}+2l_{2,1}}s_2^{l_{1,2}+2l_{2,2}}s_3^{l_{1,3}+2l_{2,3}}\binom{3}{2}-$$

$$-m_{1,1}m_{1,2}t_1^{m_{1,1}-1}t_2^{m_{1,2}-1}t_3^{m_{1,3}}s_1^{l_{1,1}+3l_{2,1}}s_2^{l_{1,2}+3l_{2,2}}s_3^{l_{1,3}+3l_{2,3}}\binom{3}{3}$$

$t_1=t_2=t_3=s_1=s_2=s_3=1$

$$[m_{1,1}+3m_{2,1}][m_{1,2}+3m_{2,2}]\binom{3}{0}-[m_{1,1}+2m_{2,1}][m_{1,2}+2m_{2,2}]\binom{3}{1}+[m_{1,1}+m_{2,1}][m_{1,2}+m_{2,2}]\binom{3}{2}-m_{1,1}m_{1,2}\binom{3}{3}=$$

$$=[m_{1,1}+3m_{2,1}][m_{1,2}+3m_{2,2}]-[m_{1,1}+2m_{2,1}][m_{1,2}+2m_{2,2}]3+[m_{1,1}+m_{2,1}][m_{1,2}+m_{2,2}]3-m_{1,1}m_{1,2}=0$$

Deriváljunk előbb t_1, majd t_3 szerint!

$$[m_{1,1}+3m_{2,1}][m_{1,3}+3m_{2,3}]t_1^{m_{1,1}+3m_{2,1}-1}t_2^{m_{1,2}+3m_{2,2}}t_3^{m_{1,3}+3m_{2,3}-1}s_1^{l_{1,1}}s_2^{l_{1,2}}s_3^{l_{1,3}}\binom{3}{0}-$$

$$-[m_{1,1}+2m_{2,1}][m_{1,3}+2m_{2,3}]t_1^{m_{1,1}+2m_{2,1}-1}t_2^{m_{1,2}+2m_{2,2}}t_3^{m_{1,3}+2m_{2,3}-1}s_1^{l_{1,1}+l_{2,1}}s_2^{l_{1,2}+l_{2,2}}s_3^{l_{1,3}+l_{2,3}}\binom{3}{1}+$$

$$+[m_{1,1}+m_{2,1}][m_{1,3}+m_{2,3}]t_1^{m_{1,1}+m_{2,1}-1}t_2^{m_{1,2}+m_{2,2}}t_3^{m_{1,3}+m_{2,3}-1}s_1^{l_{1,1}+2l_{2,1}}s_2^{l_{1,2}+2l_{2,2}}s_3^{l_{1,3}+2l_{2,3}}\binom{3}{2}-$$

$$-m_{1,1}m_{1,3}t_1^{m_{1,1}-1}t_2^{m_{1,2}}t_3^{m_{1,3}-1}s_1^{l_{1,1}+3l_{2,1}}s_2^{l_{1,2}+3l_{2,2}}s_3^{l_{1,3}+3l_{2,3}}\binom{3}{3}$$

$t_1=t_2=t_3=s_1=s_2=s_3=1$

$$[m_{1,1}+3m_{2,1}][m_{1,3}+3m_{2,3}]\binom{3}{0}-[m_{1,1}+2m_{2,1}][m_{1,3}+2m_{2,3}]\binom{3}{1}+[m_{1,1}+m_{2,1}][m_{1,3}+m_{2,3}]\binom{3}{2}-m_{1,1}m_{1,3}\binom{3}{3}=$$

$$=[m_{1,1}+3m_{2,1}][m_{1,3}+3m_{2,3}]-[m_{1,1}+2m_{2,1}][m_{1,3}+2m_{2,3}]3+[m_{1,1}+m_{2,1}][m_{1,3}+m_{2,3}]3-m_{1,1}m_{1,3}=0$$

Deriváljunk előbb t_1, majd t_3 szerint!

$$[m_{1,2}+3m_{2,2}][m_{1,3}+3m_{2,3}]t_1^{m_{1,1}+3m_{2,1}}t_2^{m_{1,2}+3m_{2,2}-1}t_3^{m_{1,3}+3m_{2,3}-1}s_1^{l_{1,1}}s_2^{l_{1,2}}s_3^{l_{1,3}}\binom{3}{0}-$$

$$-[m_{1,2}+2m_{2,2}][m_{1,3}+2m_{2,3}]t_1^{m_{1,1}+2m_{2,1}}t_2^{m_{1,2}+2m_{2,2}-1}t_3^{m_{1,3}+2m_{2,3}-1}s_1^{l_{1,1}+l_{2,1}}s_2^{l_{1,2}+l_{2,2}}s_3^{l_{1,3}+l_{2,3}}\binom{3}{1}+$$

$$+[m_{1,2}+m_{2,2}][m_{1,3}+m_{2,3}]t_1^{m_{1,1}+m_{2,1}}t_2^{m_{1,2}+m_{2,2}-1}t_3^{m_{1,3}+m_{2,3}-1}s_1^{l_{1,1}+2l_{2,1}}s_2^{l_{1,2}+2l_{2,2}}s_3^{l_{1,3}+2l_{2,3}}\binom{3}{2}-$$

$$-m_{1,2}m_{1,3}t_1^{m_{1,1}}t_2^{m_{1,2}-1}t_3^{m_{1,3}-1}s_1^{l_{1,1}+3l_{2,1}}s_2^{l_{1,2}+3l_{2,2}}s_3^{l_{1,3}+3l_{2,3}}\binom{3}{3}$$

$t_1=t_2=t_3=s_1=s_2=s_3=1$

$$[m_{1,2}+3m_{2,2}][m_{1,3}+3m_{2,3}]\binom{3}{0}-[m_{1,2}+2m_{2,2}][m_{1,3}+2m_{2,3}]\binom{3}{1}+[m_{1,2}+m_{2,2}][m_{1,3}+m_{2,3}]\binom{3}{2}-m_{1,2}m_{1,3}\binom{3}{3}=$$

$$=[m_{1,2}+3m_{2,2}][m_{1,3}+3m_{2,3}]-[m_{1,2}+2m_{2,2}][m_{1,3}+2m_{2,3}]3+[m_{1,2}+m_{2,2}][m_{1,3}+m_{2,3}]3-m_{1,2}m_{1,3}=0$$

Deriváljunk t_1,t_2,t_3 szerint!

$$[m_{1,1}+3m_{2,1}][m_{1,2}+3m_{2,2}][m_{1,3}+3m_{2,3}]t_1^{m_{1,1}+3m_{2,1}-1}t_2^{m_{1,2}+3m_{2,2}-1}t_3^{m_{1,3}+3m_{2,3}-1}s_1^{l_{1,1}}s_2^{l_{1,2}}s_3^{l_{1,3}}\binom{3}{0}-$$

$$-[m_{1,1}+2m_{2,1}][m_{1,2}+2m_{2,2}][m_{1,3}+2m_{2,3}]t_1^{m_{1,1}+2m_{2,1}-1}t_2^{m_{1,2}+2m_{2,2}-1}t_3^{m_{1,3}+2m_{2,3}-1}s_1^{l_{1,1}+l_{2,1}}s_2^{l_{1,2}+l_{2,2}}s_3^{l_{1,3}+l_{2,3}}\binom{3}{1}+$$

$$+[m_{1,1}+m_{2,1}][m_{1,2}+m_{2,2}][m_{1,3}+m_{2,3}]t_1^{m_{1,1}+m_{2,1}-1}t_2^{m_{1,2}+m_{2,2}-1}t_3^{m_{1,3}+m_{2,3}-1}s_1^{l_{1,1}+2l_{2,1}}s_2^{l_{1,2}+2l_{2,2}}s_3^{l_{1,3}+2l_{2,3}}\binom{3}{2}-$$

$$-m_{1,1}m_{1,2}m_{1,3}t_1^{m_{1,1}-1}t_2^{m_{1,2}-1}t_3^{m_{1,3}-1}s_1^{l_{1,1}+3l_{2,1}}s_2^{l_{1,2}+3l_{2,2}}s_3^{l_{1,3}+3l_{2,3}}\binom{3}{3}$$

$t_1=t_2=t_3=s_1=s_2=s_3=1$

$$[m_{1,1}+3m_{2,1}][m_{1,2}+3m_{2,2}][m_{1,3}+3m_{2,3}]\binom{3}{0}-[m_{1,1}+2m_{2,1}][m_{1,2}+2m_{2,2}][m_{1,3}+2m_{2,3}]\binom{3}{1}+$$

$$+[m_{1,1}+m_{2,1}][m_{1,2}+m_{2,2}][m_{1,3}+m_{2,3}]\binom{3}{2}-m_{1,1}m_{1,2}m_{1,3}\binom{3}{3}=$$

$$=[m_{1,1}+3m_{2,1}][m_{1,2}+3m_{2,2}][m_{1,3}+3m_{2,3}]-[m_{1,1}+2m_{2,1}][m_{1,2}+2m_{2,2}][m_{1,3}+2m_{2,3}]3+$$

$$+[m_{1,1}+m_{2,1}][m_{1,2}+m_{2,2}][m_{1,3}+m_{2,3}]3-m_{1,1}m_{1,2}m_{1,3}=3!m_{2,1}m_{2,2}m_{2,3}$$

$m_{2,1}=m_{2,2}=m_{2,3}=1$, esetén 6=3!

Deriváljunk s_1,s_2,s_3 szerint!

$$l_{1,1}l_{1,2}l_{1,3}t_1^{m_{1,1}+3m_{2,1}}t_2^{m_{1,2}+3m_{2,2}}t_3^{m_{1,3}+3m_{2,3}}s_1^{l_{1,1}-1}s_2^{l_{1,2}-1}s_3^{l_{1,3}-1}\binom{3}{0}-$$

$$-[l_{1,1}+l_{2,1}][l_{1,2}+l_{2,2}][l_{1,3}+l_{2,3}]t_1^{m_{1,1}+2m_{2,1}}t_2^{m_{1,2}+2m_{2,2}}t_3^{m_{1,3}+2m_{2,3}}s_1^{l_{1,1}+l_{2,1}-1}s_2^{l_{1,2}+l_{2,2}-1}s_3^{l_{1,3}+l_{2,3}-1}\binom{3}{1}+$$

$$+[l_{1,1}+2l_{2,1}][l_{1,2}+2l_{2,2}][l_{1,3}+2l_{2,3}]t_1^{m_{1,1}+m_{2,1}}t_2^{m_{1,2}+m_{2,2}}t_3^{m_{1,3}+m_{2,3}}s_1^{l_{1,1}+2l_{2,1}-1}s_2^{l_{1,2}+2l_{2,2}-1}s_3^{l_{1,3}+2l_{2,3}-1}\binom{3}{2}-$$

$$-[l_{1,1}+3l_{2,1}][l_{1,2}+3l_{2,2}][l_{1,3}+3l_{2,3}]t_1^{m_{1,1}}t_2^{m_{1,2}}t_3^{m_{1,3}}s_1^{l_{1,1}+3l_{2,1}-1}s_2^{l_{1,2}+3l_{2,2}-1}s_3^{l_{1,3}+3l_{2,3}-1}\binom{3}{3}$$

$t_1=t_2=t_3=s_1=s_2=s_3=1$

$$l_{1,1}l_{1,2}l_{1,3}\binom{3}{0}-[l_{1,1}+l_{2,1}][l_{1,2}+l_{2,2}][l_{1,3}+l_{2,3}]\binom{3}{1}+$$

$$+[l_{1,1}+2l_{2,1}][l_{1,2}+2l_{2,2}][l_{1,3}+2l_{2,3}]\binom{3}{2}-[l_{1,1}+3l_{2,1}][l_{1,2}+3l_{2,2}][l_{1,3}+3l_{2,3}]\binom{3}{3}=$$

$$=l_{1,1}l_{1,2}l_{1,3}-[l_{1,1}+l_{2,1}][l_{1,2}+l_{2,2}][l_{1,3}+l_{2,3}]3+$$

$$+[l_{1,1}+2l_{2,1}][l_{1,2}+2l_{2,2}][l_{1,3}+2l_{2,3}]3-[l_{1,1}+3l_{2,1}][l_{1,2}+3l_{2,2}][l_{1,3}+3l_{2,3}]=-3!l_{2,1}l_{2,2}l_{2,3}$$

$l_{2,1}=l_{2,2}=l_{2,3}=1$, esetén -6=-3!

Deriváljunk t_1,t_2,t_3,s_1 szerint!

$$[m_{1,1}+3m_{2,1}][m_{1,2}+3m_{2,2}][m_{1,3}+3m_{2,3}]l_{1,1}t_1^{m_{1,1}+3m_{2,1}-1}t_2^{m_{1,2}+3m_{2,2}-1}t_3^{m_{1,3}+3m_{2,3}-1}s_1^{l_{1,1}-1}s_2^{l_{1,2}}s_3^{l_{1,3}}\binom{3}{0}-$$

$$-[m_{1,1}+2m_{2,1}][m_{1,2}+2m_{2,2}][m_{1,3}+2m_{2,3}][l_{1,1}+l_{2,1}]t_1^{m_{1,1}+2m_{2,1}-1}t_2^{m_{1,2}+2m_{2,2}-1}t_3^{m_{1,3}+2m_{2,3}-1}s_1^{l_{1,1}+l_{2,1}-1}s_2^{l_{1,2}+l_{2,2}}s_3^{l_{1,3}+l_{2,3}}\binom{3}{1}+$$

$$+[m_{1,1}+m_{2,1}][m_{1,2}+m_{2,2}][m_{1,3}+m_{2,3}][l_{1,1}+2l_{2,1}]t_1^{m_{1,1}+m_{2,1}-1}t_2^{m_{1,2}+m_{2,2}-1}t_3^{m_{1,3}+m_{2,3}-1}s_1^{l_{1,1}+2l_{2,1}-1}s_2^{l_{1,2}+2l_{2,2}}s_3^{l_{1,3}+2l_{2,3}}\binom{3}{2}-$$

$$-m_{1,1}m_{1,2}m_{1,3}[l_{1,1}+3l_{2,1}]t_1^{m_{1,1}-1}t_2^{m_{1,2}-1}t_3^{m_{1,3}-1}s_1^{l_{1,1}+3l_{2,1}-1}s_2^{l_{1,2}+3l_{2,2}}s_3^{l_{1,3}+3l_{2,3}}\binom{3}{3}$$

$t_1=t_2=t_3=s_1=s_2=s_3=1$

$$[m_{1,1}+3m_{2,1}][m_{1,2}+3m_{2,2}][m_{1,3}+3m_{2,3}]l_{1,1}\binom{3}{0}-[m_{1,1}+2m_{2,1}][m_{1,2}+2m_{2,2}][m_{1,3}+2m_{2,3}][l_{1,1}+l_{2,1}]\binom{3}{1}+$$

$$+[m_{1,1}+m_{2,1}][m_{1,2}+m_{2,2}][m_{1,3}+m_{2,3}][l_{1,1}+2l_{2,1}]\binom{3}{2}-m_{1,1}m_{1,2}m_{1,3}[l_{1,1}+3l_{2,1}]\binom{3}{3}=$$

$$=[m_{1,1}+3m_{2,1}][m_{1,2}+3m_{2,2}][m_{1,3}+3m_{2,3}]l_{1,1}-[m_{1,1}+2m_{2,1}][m_{1,2}+2m_{2,2}][m_{1,3}+2m_{2,3}][l_{1,1}+l_{2,1}]3+$$

$$+[m_{1,1}+m_{2,1}][m_{1,2}+m_{2,2}][m_{1,3}+m_{2,3}][l_{1,1}+2l_{2,1}]3-m_{1,1}m_{1,2}m_{1,3}[l_{1,1}+3l_{2,1}]<0$$

$m_{1,1}=m_{1,2}=m_{1,3}=m_{2,1}=m_{2,2}=m_{2,3}=l_{1,1}=l_{1,2}=l_{1,3}=l_{2,1}=l_{2,2}=l_{2,3}=1$, esetén -30

Deriváljunk t_1,t_2,t_3,s_1,s_2 szerint!

$$[m_{1,1}+3m_{2,1}][m_{1,2}+3m_{2,2}][m_{1,3}+3m_{2,3}]l_{1,1}l_{1,2}t_1^{m_{1,1}+3m_{2,1}-1}t_2^{m_{1,2}+3m_{2,2}-1}t_3^{m_{1,3}+3m_{2,3}-1}s_1^{l_{1,1}-1}s_2^{l_{1,2}-1}s_3^{l_{1,3}}\binom{3}{0}-$$

$$-[m_{1,1}+2m_{2,1}][m_{1,2}+2m_{2,2}][m_{1,3}+2m_{2,3}][l_{1,1}+l_{2,1}][l_{1,2}+l_{2,2}]\cdot$$

$$\cdot t_1^{m_{1,1}+2m_{2,1}-1}t_2^{m_{1,2}+2m_{2,2}-1}t_3^{m_{1,3}+2m_{2,3}-1}s_1^{l_{1,1}+l_{2,1}-1}s_2^{l_{1,2}+l_{2,2}-1}s_3^{l_{1,3}+l_{2,3}}\binom{3}{1}+$$

$$+[m_{1,1}+m_{2,1}][m_{1,2}+m_{2,2}][m_{1,3}+m_{2,3}][l_{1,1}+2l_{2,1}][l_{1,2}+2l_{2,2}]\cdot$$

$$\cdot t_1^{m_{1,1}+m_{2,1}-1}t_2^{m_{1,2}+m_{2,2}-1}t_3^{m_{1,3}+m_{2,3}-1}s_1^{l_{1,1}+2l_{2,1}-1}s_2^{l_{1,2}+2l_{2,2}-1}s_3^{l_{1,3}+2l_{2,3}}\binom{3}{2}-$$

$$-m_{1,1}m_{1,2}m_{1,3}[l_{1,1}+3l_{2,1}][l_{1,2}+3l_{2,2}]t_1^{m_{1,1}-1}t_2^{m_{1,2}-1}t_3^{m_{1,3}-1}s_1^{l_{1,1}+3l_{2,1}-1}s_2^{l_{1,2}+3l_{2,2}-1}s_3^{l_{1,3}+3l_{2,3}}\binom{3}{3}$$

$t_1=t_2=t_3=s_1=s_2=s_3=1$

$$[m_{1,1}+3m_{2,1}][m_{1,2}+3m_{2,2}][m_{1,3}+3m_{2,3}]l_{1,1}l_{1,2}\binom{3}{0}-$$

$$-[m_{1,1}+2m_{2,1}][m_{1,2}+2m_{2,2}][m_{1,3}+2m_{2,3}][l_{1,1}+l_{2,1}][l_{1,2}+l_{2,2}]\binom{3}{1}+$$

$$+[m_{1,1}+m_{2,1}][m_{1,2}+m_{2,2}][m_{1,3}+m_{2,3}][l_{1,1}+2l_{2,1}][l_{1,2}+2l_{2,2}]\binom{3}{2}-m_{1,1}m_{1,2}m_{1,3}[l_{1,1}+3l_{2,1}][l_{1,2}+3l_{2,2}]\binom{3}{3}=$$

$$=[m_{1,1}+3m_{2,1}][m_{1,2}+3m_{2,2}][m_{1,3}+3m_{2,3}]l_{1,1}l_{1,2}-[m_{1,1}+2m_{2,1}][m_{1,2}+2m_{2,2}][m_{1,3}+2m_{2,3}][l_{1,1}+l_{2,1}][l_{1,2}+l_{2,2}]3+$$

$$+[m_{1,1}+m_{2,1}][m_{1,2}+m_{2,2}][m_{1,3}+m_{2,3}][l_{1,1}+2l_{2,1}][l_{1,2}+2l_{2,2}]3-m_{1,1}m_{1,2}m_{1,3}[l_{1,1}+3l_{2,1}][l_{1,2}+3l_{2,2}]<0$$

$m_{1,1}=m_{1,2}=m_{1,3}=m_{2,1}=m_{2,2}=m_{2,3}=l_{1,1}=l_{1,2}=l_{1,3}=l_{2,1}=l_{2,2}=l_{2,3}=1$, esetén -60

Deriváljunk t_1,t_2,t_3,s_1,s_2,s_3 szerint!

$$[m_{1,1}+3m_{2,1}][m_{1,2}+3m_{2,2}][m_{1,3}+3m_{2,3}]l_{1,1}l_{1,2}l_{1,3}t_1^{m_{1,1}+3m_{2,1}-1}t_2^{m_{1,2}+3m_{2,2}-1}t_3^{m_{1,3}+3m_{2,3}-1}s_1^{l_{1,1}-1}s_2^{l_{1,2}-1}s_3^{l_{1,3}-1}\binom{3}{0}-$$

$$-[m_{1,1}+2m_{2,1}][m_{1,2}+2m_{2,2}][m_{1,3}+2m_{2,3}][l_{1,1}+l_{2,1}][l_{1,2}+l_{2,2}][l_{1,3}+l_{2,3}]\cdot$$

$$\cdot t_1^{m_{1,1}+2m_{2,1}-1}t_2^{m_{1,2}+2m_{2,2}-1}t_3^{m_{1,3}+2m_{2,3}-1}s_1^{l_{1,1}+l_{2,1}-1}s_2^{l_{1,2}+l_{2,2}-1}s_3^{l_{1,3}+l_{2,3}-1}\binom{3}{1}+$$

$$+[m_{1,1}+m_{2,1}][m_{1,2}+m_{2,2}][m_{1,3}+m_{2,3}][l_{1,1}+2l_{2,1}][l_{1,2}+2l_{2,2}][l_{1,3}+2l_{2,3}]\cdot$$

$$\cdot t_1^{m_{1,1}+m_{2,1}-1}t_2^{m_{1,2}+m_{2,2}-1}t_3^{m_{1,3}+m_{2,3}-1}s_1^{l_{1,1}+2l_{2,1}-1}s_2^{l_{1,2}+2l_{2,2}-1}s_3^{l_{1,3}+2l_{2,3}-1}\binom{3}{2}-$$

$$-m_{1,1}m_{1,2}m_{1,3}[l_{1,1}+3l_{2,1}][l_{1,2}+3l_{2,2}][l_{1,3}+3l_{2,3}]t_1^{m_{1,1}-1}t_2^{m_{1,2}-1}t_3^{m_{1,3}-1}s_1^{l_{1,1}+3l_{2,1}-1}s_2^{l_{1,2}+3l_{2,2}-1}s_3^{l_{1,3}+3l_{2,3}-1}\binom{3}{3}$$

$t_1=t_2=t_3=s_1=s_2=s_3=1$

$$[m_{1,1}+3m_{2,1}][m_{1,2}+3m_{2,2}][m_{1,3}+3m_{2,3}]l_{1,1}l_{1,2}l_{1,3}\binom{3}{0}-$$

$$-[m_{1,1}+2m_{2,1}][m_{1,2}+2m_{2,2}][m_{1,3}+2m_{2,3}][l_{1,1}+l_{2,1}][l_{1,2}+l_{2,2}][l_{1,3}+l_{2,3}]\binom{3}{1}+$$

$$+[m_{1,1}+m_{2,1}][m_{1,2}+m_{2,2}][m_{1,3}+m_{2,3}][l_{1,1}+2l_{2,1}][l_{1,2}+2l_{2,2}][l_{1,3}+2l_{2,3}]\binom{3}{2}-$$

$$-m_{1,1}m_{1,2}m_{1,3}[l_{1,1}+3l_{2,1}][l_{1,2}+3l_{2,2}][l_{1,3}+3l_{2,3}]\binom{3}{3}=$$

$$=[m_{1,1}+3m_{2,1}][m_{1,2}+3m_{2,2}][m_{1,3}+3m_{2,3}]l_{1,1}l_{1,2}l_{1,3}-$$

$$-[m_{1,1}+2m_{2,1}][m_{1,2}+2m_{2,2}][m_{1,3}+2m_{2,3}][l_{1,1}+l_{2,1}][l_{1,2}+l_{2,2}][l_{1,3}+l_{2,3}]3+$$

$$+[m_{1,1}+m_{2,1}][m_{1,2}+m_{2,2}][m_{1,3}+m_{2,3}][l_{1,1}+2l_{2,1}][l_{1,2}+2l_{2,2}][l_{1,3}+2l_{2,3}]3-$$

$$-m_{1,1}m_{1,2}m_{1,3}[l_{1,1}+3l_{2,1}][l_{1,2}+3l_{2,2}][l_{1,3}+3l_{2,3}]=0$$

$m_{1,1}=m_{1,2}=m_{1,3}=m_{2,1}=m_{2,2}=m_{2,3}=l_{1,1}=l_{1,2}=l_{1,3}=l_{2,1}=l_{2,2}=l_{2,3}=1$, esetén 0

$n=4$

$$t_1^{m_{1,1}}t_2^{m_{1,2}}t_3^{m_{1,3}}t_4^{m_{1,4}}s_1^{l_{1,1}}s_2^{l_{1,2}}s_3^{l_{1,3}}s_4^{m_{1,4}}(t_1^{m_{2,1}}t_2^{m_{2,2}}t_3^{m_{2,3}}t_4^{m_{2,4}}-s_1^{l_{2,1}}s_2^{l_{2,2}}s_3^{l_{2,3}}s_4^{m_{2,4}})^4=$$

$$=t_1^{m_{1,1}+4m_{2,1}}t_2^{m_{1,2}+4m_{2,2}}t_3^{m_{1,3}+4m_{2,3}}t_4^{m_{1,3}+4m_{2,4}}s_1^{l_{1,1}}s_2^{l_{1,2}}s_3^{l_{1,3}}s_4^{l_{1,4}}\binom{4}{0}-$$

$$-t_1^{m_{1,1}+3m_{2,1}}t_2^{m_{1,2}+3m_{2,2}}t_3^{m_{1,3}+3m_{2,3}}t_4^{m_{1,3}+3m_{2,4}}s_1^{l_{1,1}+l_{2,1}}s_2^{l_{1,2}+l_{2,2}}s_3^{l_{1,2}+l_{2,3}}s_4^{l_{1,2}+l_{2,4}}\binom{4}{1}+$$

$$+t_1^{m_{1,1}+2m_{2,1}}t_2^{m_{1,2}+2m_{2,2}}t_3^{m_{1,3}+2m_{2,3}}t_4^{m_{1,3}+2m_{2,4}}s_1^{l_{1,1}+2l_{2,1}}s_2^{l_{1,2}+2l_{2,2}}s_3^{l_{1,2}+2l_{2,3}}s_4^{l_{1,2}+2l_{2,4}}\binom{4}{2}-$$

$$-t_1^{m_{1,1}+m_{2,1}}t_2^{m_{1,2}+m_{2,2}}t_3^{m_{1,3}+m_{2,3}}t_4^{m_{1,3}+m_{2,4}}s_1^{l_{1,1}+3l_{2,1}}s_2^{l_{1,2}+3l_{2,2}}s_3^{l_{1,2}+3l_{2,3}}s_4^{l_{1,2}+3l_{2,4}}\binom{4}{3}+$$

$$+t_1^{m_{1,1}}t_2^{m_{1,2}}t_3^{m_{1,3}}t_4^{m_{1,4}}s_1^{l_{1,1}+4l_{2,1}}s_2^{l_{1,2}+4l_{2,2}}s_3^{l_{1,2}+4l_{2,3}}s_4^{l_{1,2}+4l_{2,4}}\binom{4}{4}$$

t_1, t_2, t_3, t_4 -szerint deriválunk!

$$4! m_{2,1} m_{2,2} m_{2,3} m_{2,4} =$$

$$= [m_{1,1} + 4m_{2,1}][m_{1,2} + 4m_{2,2}][m_{1,3} + 4m_{2,3}][m_{1,4} + 4m_{2,4}] \cdot$$

$$\cdot t_1^{m_{1,1}+4m_{2,1}-1} t_2^{m_{1,2}+4m_{2,2}-1} t_3^{m_{1,3}+4m_{2,3}-1} t_4^{m_{1,3}+4m_{2,4}-1} s_1^{l_{1,1}} s_2^{l_{1,2}} s_3^{l_{1,3}} s_4^{l_{1,4}} \binom{4}{0} -$$

$$- [m_{1,1} + 3m_{2,1}][m_{1,2} + 3m_{2,2}][m_{1,3} + 3m_{2,3}][m_{1,4} + 3m_{2,4}] \cdot$$

$$\cdot t_1^{m_{1,1}+3m_{2,1}-1} t_2^{m_{1,2}+3m_{2,2}-1} t_3^{m_{1,3}+3m_{2,3}-1} t_4^{m_{1,3}+3m_{2,4}-1} s_1^{l_{1,1}+l_{2,1}} s_2^{l_{1,2}+l_{2,2}} s_3^{l_{1,2}+l_{2,3}} s_4^{l_{1,2}+l_{2,4}} \binom{4}{1} +$$

$$+ [m_{1,1} + 2m_{2,1}][m_{1,2} + 2m_{2,2}][m_{1,3} + 2m_{2,3}][m_{1,4} + 2m_{2,4}] \cdot$$

$$\cdot t_1^{m_{1,1}+2m_{2,1}-1} t_2^{m_{1,2}+2m_{2,2}-1} t_3^{m_{1,3}+2m_{2,3}-1} t_4^{m_{1,3}+2m_{2,4}-1} s_1^{l_{1,1}+2l_{2,1}} s_2^{l_{1,2}+2l_{2,2}} s_3^{l_{1,2}+2l_{2,3}} s_4^{l_{1,2}+2l_{2,4}} \binom{4}{2} -$$

$$- [m_{1,1} + m_{2,1}][m_{1,2} + m_{2,2}][m_{1,3} + m_{2,3}][m_{1,4} + m_{2,4}] \cdot$$

$$\cdot t_1^{m_{1,1}+m_{2,1}-1} t_2^{m_{1,2}+m_{2,2}-1} t_3^{m_{1,3}+m_{2,3}-1} t_4^{m_{1,3}+m_{2,4}-1} s_1^{l_{1,1}+3l_{2,1}} s_2^{l_{1,2}+3l_{2,2}} s_3^{l_{1,2}+3l_{2,3}} s_4^{l_{1,2}+3l_{2,4}} \binom{4}{3} +$$

$$+ m_{1,1} m_{1,2} m_{1,3} m_{1,4} t_1^{m_{1,1}-1} t_2^{m_{1,2}-1} t_3^{m_{1,3}-1} t_4^{m_{1,4}-1} s_1^{l_{1,1}+4l_{2,1}} s_2^{l_{1,2}+4l_{2,2}} s_3^{l_{1,2}+4l_{2,3}} s_4^{l_{1,2}+4l_{2,4}} \binom{4}{4}$$

$t_1, t_2, t_3, t_4, s_1, s_2, s_3, s_4 = 1$

$$[m_{1,1} + 4m_{2,1}][m_{1,2} + 4m_{2,2}][m_{1,3} + 4m_{2,3}][m_{1,4} + 4m_{2,4}] \binom{4}{0} -$$

$$- [m_{1,1} + 3m_{2,1}][m_{1,2} + 3m_{2,2}][m_{1,3} + 3m_{2,3}][m_{1,4} + 3m_{2,4}] \binom{4}{1} +$$

$$+ [m_{1,1} + 2m_{2,1}][m_{1,2} + 2m_{2,2}][m_{1,3} + 2m_{2,3}][m_{1,4} + 2m_{2,4}] \binom{4}{2} -$$

$$- [m_{1,1} + m_{2,1}][m_{1,2} + m_{2,2}][m_{1,3} + m_{2,3}][m_{1,4} + m_{2,4}] \binom{4}{3} + m_{1,1} m_{1,2} m_{1,3} m_{1,4} \binom{4}{4} = 4! m_{2,1} m_{2,2} m_{2,3} m_{2,4}$$

$m_{2,1} = m_{2,2} = m_{2,3} = m_{2,4} = 1$ esetén 4!

s_2, s_3, s_4-szerint deriválunk!

$$4! l_{2,1} l_{2,2} l_{2,3} l_{2,4} =$$

$$= l_{1,1} l_{1,2} l_{1,3} l_{1,4} t_1^{m_{1,1}+4m_{2,1}} t_2^{m_{1,2}+4m_{2,2}} t_3^{m_{1,3}+4m_{2,3}} t_4^{m_{1,3}+4m_{2,4}} s_1^{l_{1,1}-1} s_2^{l_{1,2}-1} s_3^{l_{1,3}} s_4^{l_{1,4}-1} \binom{4}{0} -$$

$$- [l_{1,1} + l_{2,1}][l_{1,2} + l_{2,2}][l_{1,3} + l_{2,3}][l_{1,4} + l_{2,4}] \cdot$$

$$\cdot t_1^{m_{1,1}+3m_{2,1}} t_2^{m_{1,2}+3m_{2,2}} t_3^{m_{1,3}+3m_{2,3}} t_4^{m_{1,3}+3m_{2,4}} s_1^{l_{1,1}+l_{2,1}-1} s_2^{l_{1,2}+l_{2,2}-1} s_3^{l_{1,2}+l_{2,3}-1} s_4^{l_{1,2}+l_{2,4}-1} \binom{4}{1} +$$

$$+ [l_{1,1} + 2l_{2,1}][l_{1,2} + 2l_{2,2}][l_{1,3} + 2l_{2,3}][l_{1,4} + 2l_{2,4}] \cdot$$

$$\cdot t_1^{m_{1,1}+2m_{2,1}} t_2^{m_{1,2}+2m_{2,2}} t_3^{m_{1,3}+2m_{2,3}} t_4^{m_{1,3}+2m_{2,4}} s_1^{l_{1,1}+2l_{2,1}-1} s_2^{l_{1,2}+2l_{2,2}-1} s_3^{l_{1,2}+2l_{2,3}-1} s_4^{l_{1,2}+2l_{2,4}-1} \binom{4}{2} -$$

$$- [l_{1,1} + 3l_{2,1}][l_{1,2} + 3l_{2,2}][l_{1,3} + 3l_{2,3}][l_{1,4} + 3l_{2,4}] \cdot$$

$$\cdot t_1^{m_{1,1}+m_{2,1}} t_2^{m_{1,2}+m_{2,2}} t_3^{m_{1,3}+m_{2,3}} t_4^{m_{1,3}+m_{2,4}} s_1^{l_{1,1}+3l_{2,1}-1} s_2^{l_{1,2}+3l_{2,2}-1} s_3^{l_{1,2}+3l_{2,3}} s_4^{l_{1,2}+3l_{2,4}-1} \binom{4}{3} +$$

$$+ [l_{1,1} + 4l_{2,1}][l_{1,2} + 4l_{2,2}][l_{1,3} + 4l_{2,3}][m_{1,4} + 4l_{2,4}] \cdot$$

$$\cdot t_1^{m_{1,1}} t_2^{m_{1,2}} t_3^{m_{1,3}} t_4^{m_{1,4}} s_1^{l_{1,1}+4l_{2,1}-1} s_2^{l_{1,2}+4l_{2,2}-1} s_3^{l_{1,2}+4l_{2,3}-1} s_4^{l_{1,2}+4l_{2,4}-1} \binom{4}{4}$$

$t_1,t_2,t_3,t_4,s_1,s_2,s_3,s_4=1$

$$l_{1,1}l_{1,2}l_{1,3}l_{1,4}\binom{4}{0}-[l_{1,1}+l_{2,1}][l_{1,2}+l_{2,2}][l_{1,3}+l_{2,3}][l_{1,4}+l_{2,4}]\binom{4}{1}+[l_{1,1}+2l_{2,1}][l_{1,2}+2l_{2,2}][l_{1,3}+2l_{2,3}][l_{1,4}+2l_{2,4}]\binom{4}{2}-$$

$$-[l_{1,1}+3l_{2,1}][l_{1,2}+3l_{2,2}][l_{1,3}+3l_{2,3}][l_{1,4}+3l_{2,4}]\binom{4}{3}+[l_{1,1}+4l_{2,1}][l_{1,2}+4l_{2,2}][l_{1,3}+4l_{2,3}][m_{1,4}+4l_{2,4}]\binom{4}{4}=$$

$$=l_{1,1}l_{1,2}l_{1,3}l_{1,4}-[l_{1,1}+l_{2,1}][l_{1,2}+l_{2,2}][l_{1,3}+l_{2,3}][l_{1,4}+l_{2,4}]4+[l_{1,1}+2l_{2,1}][l_{1,2}+2l_{2,2}][l_{1,3}+2l_{2,3}][l_{1,4}+2l_{2,4}]6-$$

$$-[l_{1,1}+3l_{2,1}][l_{1,2}+3l_{2,2}][l_{1,3}+3l_{2,3}][l_{1,4}+3l_{2,4}]4+[l_{1,1}+4l_{2,1}][l_{1,2}+4l_{2,2}][l_{1,3}+4l_{2,3}][m_{1,4}+4l_{2,4}]=4!l_{2,1}l_{2,2}l_{2,3}l_{2,4}$$

$l_{2,1}=l_{2,2}=l_{2,3}=l_{2,4}=1$ esetén 4!

$t_1,t_2,t_3,t_4,s_1,s_2,s_3,s_4$-szerint deriválunk!

$$[m_{1,1}+4m_{2,1}][m_{1,2}+4m_{2,2}][m_{1,3}+4m_{2,3}][m_{1,4}+4m_{2,4}]l_{1,1}l_{1,2}l_{1,3}l_{1,4}\cdot$$

$$\cdot t_1^{m_{1,1}+4m_{2,1}-1}t_2^{m_{1,2}+4m_{2,2}-1}t_3^{m_{1,3}+4m_{2,3}-1}t_4^{m_{1,3}+4m_{2,4}-1}s_1^{l_{1,1}-1}s_2^{l_{1,2}-1}s_3^{l_{1,3}}s_4^{l_{1,4}-1}\binom{4}{0}-$$

$$-[m_{1,1}+3m_{2,1}][m_{1,2}+3m_{2,2}][m_{1,3}+3m_{2,3}][m_{1,4}+3m_{2,4}][l_{1,1}+l_{2,1}][l_{1,2}+l_{2,2}][l_{1,3}+l_{2,3}][l_{1,4}+l_{2,4}]\cdot$$

$$\cdot t_1^{m_{1,1}+3m_{2,1}-1}t_2^{m_{1,2}+3m_{2,2}-1}t_3^{m_{1,3}+3m_{2,3}-1}t_4^{m_{1,3}+3m_{2,4}-1}s_1^{l_{1,1}+l_{2,1}-1}s_2^{l_{1,2}+l_{2,2}-1}s_3^{l_{1,2}+l_{2,3}-1}s_4^{l_{1,2}+l_{2,4}-1}\binom{4}{1}+$$

$$+[m_{1,1}+2m_{2,1}][m_{1,2}+2m_{2,2}][m_{1,3}+2m_{2,3}][m_{1,4}+2m_{2,4}][l_{1,1}+2l_{2,1}][l_{1,2}+2l_{2,2}][l_{1,3}+2l_{2,3}][l_{1,4}+2l_{2,4}]\cdot$$

$$\cdot t_1^{m_{1,1}+2m_{2,1}-1}t_2^{m_{1,2}+2m_{2,2}-1}t_3^{m_{1,3}+2m_{2,3}-1}t_4^{m_{1,3}+2m_{2,4}-1}s_1^{l_{1,1}+2l_{2,1}-1}s_2^{l_{1,2}+2l_{2,2}-1}s_3^{l_{1,2}+2l_{2,3}-1}s_4^{l_{1,2}+2l_{2,4}-1}\binom{4}{2}-$$

$$-[m_{1,1}+m_{2,1}][m_{1,2}+m_{2,2}][m_{1,3}+m_{2,3}][m_{1,4}+m_{2,4}][l_{1,1}+3l_{2,1}][l_{1,2}+3l_{2,2}][l_{1,3}+3l_{2,3}][l_{1,4}+3l_{2,4}]\cdot$$

$$\cdot t_1^{m_{1,1}+m_{2,1}-1}t_2^{m_{1,2}+m_{2,2}-1}t_3^{m_{1,3}+m_{2,3}-1}t_4^{m_{1,3}+m_{2,4}-1}s_1^{l_{1,1}+3l_{2,1}-1}s_2^{l_{1,2}+3l_{2,2}-1}s_3^{l_{1,2}+3l_{2,3}-1}s_4^{l_{1,2}+3l_{2,4}-1}\binom{4}{3}+$$

$$+m_{1,1}m_{1,2}m_{1,3}m_{1,4}[l_{1,1}+4l_{2,1}][l_{1,2}+4l_{2,2}][l_{1,3}+4l_{2,3}][l_{1,4}+4l_{2,4}]\cdot$$

$$\cdot t_1^{m_{1,1}-1}t_2^{m_{1,2}-1}t_3^{m_{1,3}-1}t_4^{m_{1,4}-1}s_1^{l_{1,1}+4l_{2,1}-1}s_2^{l_{1,2}+4l_{2,2}-1}s_3^{l_{1,2}+4l_{2,3}-1}s_4^{l_{1,2}+4l_{2,4}-1}\binom{4}{4}$$

$t_1,t_2,t_3,t_4,s_1,s_2,s_3,s_4=1$

$$[m_{1,1}+4m_{2,1}][m_{1,2}+4m_{2,2}][m_{1,3}+4m_{2,3}][m_{1,4}+4m_{2,4}]l_{1,1}l_{1,2}l_{1,3}l_{1,4}\binom{4}{0}-$$

$$-[m_{1,1}+3m_{2,1}][m_{1,2}+3m_{2,2}][m_{1,3}+3m_{2,3}][m_{1,4}+3m_{2,4}][l_{1,1}+l_{2,1}][l_{1,2}+l_{2,2}][l_{1,3}+l_{2,3}][l_{1,4}+l_{2,4}]\binom{4}{1}+$$

$$+[m_{1,1}+2m_{2,1}][m_{1,2}+2m_{2,2}][m_{1,3}+2m_{2,3}][m_{1,4}+2m_{2,4}][l_{1,1}+2l_{2,1}][l_{1,2}+2l_{2,2}][l_{1,3}+2l_{2,3}][l_{1,4}+2l_{2,4}]\binom{4}{2}-$$

$$-[m_{1,1}+m_{2,1}][m_{1,2}+m_{2,2}][m_{1,3}+m_{2,3}][m_{1,4}+m_{2,4}][l_{1,1}+3l_{2,1}][l_{1,2}+3l_{2,2}][l_{1,3}+3l_{2,3}][l_{1,4}+3l_{2,4}]\binom{4}{3}+$$

$$+m_{1,1}m_{1,2}m_{1,3}m_{1,4}[l_{1,1}+4l_{2,1}][l_{1,2}+4l_{2,2}][l_{1,3}+4l_{2,3}][l_{1,4}+4l_{2,4}]\binom{4}{4}=$$

$$=[m_{1,1}+4m_{2,1}][m_{1,2}+4m_{2,2}][m_{1,3}+4m_{2,3}][m_{1,4}+4m_{2,4}]l_{1,1}l_{1,2}l_{1,3}l_{1,4}-$$

$$-[m_{1,1}+3m_{2,1}][m_{1,2}+3m_{2,2}][m_{1,3}+3m_{2,3}][m_{1,4}+3m_{2,4}][l_{1,1}+l_{2,1}][l_{1,2}+l_{2,2}][l_{1,3}+l_{2,3}][l_{1,4}+l_{2,4}]4+$$

$$+[m_{1,1}+2m_{2,1}][m_{1,2}+2m_{2,2}][m_{1,3}+2m_{2,3}][m_{1,4}+2m_{2,4}][l_{1,1}+2l_{2,1}][l_{1,2}+2l_{2,2}][l_{1,3}+2l_{2,3}][l_{1,4}+2l_{2,4}]6-$$

$$-[m_{1,1}+m_{2,1}][m_{1,2}+m_{2,2}][m_{1,3}+m_{2,3}][m_{1,4}+m_{2,4}][l_{1,1}+3l_{2,1}][l_{1,2}+3l_{2,2}][l_{1,3}+3l_{2,3}][l_{1,4}+3l_{2,4}]4+$$

$$+m_{1,1}m_{1,2}m_{1,3}m_{1,4}[l_{1,1}+4l_{2,1}][l_{1,2}+4l_{2,2}][l_{1,3}+4l_{2,3}][l_{1,4}+4l_{2,4}]>0$$

$m_{2,1}=m_{2,2}=m_{2,3}=m_{2,4}=l_{2,1}=l_{2,2}=l_{2,3}=l_{2,4}=1$ esetén 8854

Tétel 1

A **Tétel 0** $m_{1,i}=m_i$, $l_{1,i}=l_i$, $m_{2,i}=l_{2,i}=1$, $i=1,2,3,\dots,n$ estén!

$$(m_1+n)(m_2+n)(m_3+n)\dots(m_i+n)l_1l_2l_3\dots l_j\binom{n}{0}-$$

$$-(m_1+n-1)(m_2+n-1)(m_3+n-1)\dots(m_i+n-1)(l_1+1)(l_2+1)(l_3+1)\dots(l_j+1)\binom{n}{1}+$$

$$+(m_1+n-2)(m_2+n-2)(m_3+n-2)\dots(m_i+n-2)(l_1+2)(l_2+2)(l_3+2)\dots(l_j+2)\binom{n}{2}-$$

$$-\dots\pm m_1m_2m_3\dots m_i(l_1+n)(l_2+n)(l_3+n)\dots(l_j+n)\binom{n}{n}=$$

$$=\begin{cases} 0,\ ha\ i+j<n \\ -n!,\ ha\ i+j=n,\ j=2k-1,\ k=1,2,3,\dots \\ n!,\ ha\ i+j=n,\ j=2k,\ k=1,2,3,\dots \\ <0,\ ha\ n<i+j<2n,\ n=2k,\ k=1,2,3,\dots \\ <0,\ ha\ n<i+j<2n,\ n=2k-1,\ k=1,2,3,\dots \\ >0,\ ha\ n<i+j<2n,\ n=2k-1,\ j=2k-1,\ k=1,2,3,\dots \\ 0,\ ha\ i+j=2n,\ n=1,3,5,\dots \\ <0,\ ha\ i+j=2n,\ n=2k-1,\ k=1,2,3,\dots \\ >0,\ ha\ i+j=2n,\ n=2k,\ k=1,2,3,\dots \end{cases}$$

Bizonyítás

Amennyiben a

$$t_1^{m_1}t_2^{m_2}t_3^{m_3}\dots t_n^{m_n}s_1^{l_1}s_2^{l_2}s_3^{l_3}\dots s_n^{l_n}(t_1t_2t_3\dots t_n-s_1s_2s_3\dots s_n)^n=t_1^{m_1+n}t_2^{m_2+n}t_3^{m_3+n}\dots t_n^{m_n+n}s_1^{l_1}s_2^{l_2}s_3^{l_3}\dots s_n^{l_n}\binom{n}{0}-$$

$$-t_1^{m_1+n-1}t_2^{m_2+n-1}t_3^{m_3+n-1}\dots t_n^{m_n+n-1}s_1^{l_1+1}s_2^{l_2+1}s_3^{l_3+1}\dots s_n^{l_n+1}\binom{n}{1}+t_1^{m_1+n-2}t_2^{m_2+n-2}t_3^{m_3+n-2}\dots t_n^{m_n+n-2}s_1^{l_1+2}s_2^{l_2+2}s_3^{l_3+2}\dots s_n^{l_n+2}\binom{n}{2}-$$

$$-\dots\pm t_1^{m_1}t_2^{m_2}t_3^{m_3}\dots t_n^{m_n}s_1^{l_1+n}s_2^{l_2+n}s_3^{l_3+n}\dots s_n^{l_n+n}\binom{n}{n}$$

azonosságot deriváljuk t_i, és s_j, $i,j=1,2,3,\dots,n$ változók szerint, és minden deriválás után a t_i, és s_j, $i,j=1,2,3,\dots,n$ változók értékét egyenlővé tesszük 1-el, adódik az állítás. A baloldalon a deriválások után minden tag tartalmaz $(t_1t_2t_3\dots t_n-s_1s_2s_3\dots s_n)^k$, $k=1,2,3,\dots,n$, addig, míg az $i+j$ el nem éri az n-et, így a bal oldal 0, következésképp a jobb oldal is 0. Amennyiben a deriválások száma legalább n, és legfeljebb 2n, a bal oldalon szerepelnek olyan tagok, amelyek nem tartalmaznak $(t_1t_2t_3\dots t_n-s_1s_2s_3\dots s_n)$ tényezőt, ebből az is következik, hogy a bal oldal nem egyenlő 0-val, következésképpen a jobb oldal sem egyenlő 0-val.

Tétel 2

$$(m_1+n)(m_2+n)(m_3+n)\dots(m_i+n)\binom{n}{0}-(m_1+n-1)(m_2+n-1)(m_3+n-1)\dots(m_i+n-1)\binom{n}{1}+$$

$$+(m_1+n-2)(m_2+n-2)(m_3+n-2)\dots(m_i+n-2)\binom{n}{2}-\dots\pm m_1m_2m_3\dots m_i\binom{n}{n}=\begin{cases}0,\ ha\ i<n \\ n!,\ ha\ i=n\end{cases}$$

Bizonyítás

A **Tétel 1** bizonyításánál felírt azonosságból következik abban az esetben, ha előbb a t_i, i =1,2,3,…,n változók szerint deriválunk,és a t_i, s_j, i,j=1,2,3,…,n változók értékét egyenlővé tesszük 1-el.

Tétel 3

Abban az esetben, ha előbb az s_j, j =1,2,3,…,n változók szerint deriválunk,és a t_i, s_j, i,j=1,2,3,…,n változók értékét egyenlővé tesszük 1-el, a következő azonossághoz jutunk

$$l_1 l_2 l_3 \ldots l_j \binom{n}{0} - (l_1+1)(l_2+1)(l_3+1)\ldots(l_j+1)\binom{n}{1} + (l_1+2)(l_2+2)(l_3+2)\ldots(l_j+2)\binom{n}{2} -$$

$$- \ldots \pm (l_1+n)(l_2+n)(l_3+n)\ldots(l_j+n)\binom{n}{n} = \begin{cases} 0,\ ha\ j<n \\ -n!,\ ha\ j=n,\ n=2k-1,\ k=1,2,3,\ldots \\ n!,\ ha\ j=n,\ n=2k,\ k=1,2,3,\ldots \end{cases}$$

Bizonyítás

A **Tétel 1** bizonyításánál felírt azonosságból következik abban az esetben, ha előbb az s_j, j =1,2,3,…,n változók szerint deriválunk,és a t_i, s_j, i,j=1,2,3,…,n változók értékét egyenlővé tesszük 1-el.

Tétel 4

$$(m+n)(m+n-1)(m+n-2)\ldots(n+m+1-i)l(l-1)(l-2)\ldots(l-j+1)\binom{n}{0} -$$

$$-(m+n-1)(m+n-2)\ldots(n+m-1-i)(l+1)l(l-1)\ldots(l-j+2)\binom{n}{1} +$$

$$+(m+n-2)(m+n-3)\ldots(n+m-2-i)(l+2)(l+1)l(l-1)\ldots(l-j+3)\binom{n}{2} -$$

$$- \ldots \pm m(m-1)(m-2)\ldots(m-i+1)(l+n)(l+n-1)(l+n-2)\ldots(n+l+1-j)\binom{n}{n} =$$

$$= \begin{cases} 0,\ ha\ i+j<n \\ <0,\ ha\ n \le i+j \le 2n,\ j=2k-1,\ k=1,2,3,\ldots \\ >0,\ ha\ n \le i+j \le 2n,\ j=2k,\ k=1,2,3,\ldots \end{cases} \quad ,\ m,l \ge n$$

Bizonyítás

Deriváljuk a következő azonosságot

$$t^m s^l (t-s)^n = t^{n+m} s^l \binom{n}{0} - t^{n+m-1} s^{l+1}\binom{n}{1} + t^{n+m-2} s^{l+2}\binom{n}{2} - \ldots \pm t^{m+1} s^{l+n-1}\binom{n}{n-1} \mp t^m s^{l+n}\binom{n}{n}$$

a t változó szerint i-szer, majd az s változó szerint j-szer, és tegyük egyenlővé a t, illetve az s változó értékét 1-el, ezek után adódik az állítás. Abban az esetben, ha a deriválások száma n alatt marad a bal oldal 0, ugyanis minden tag tartalmaz egy $(t-s)^k$, k=1,2,3,…,n, tényezőt. Abban az esetben viszont, ha deriválások száma legalább n, és legfeljebb 2n közzé esik, a baldal tartalmaz olyan tagokat, amelyekben nem szerepel $(t-s)^k$, k=1,2,3,…,n, tényező.

Tétel 5

$$(m+n)(m+n-1)(m+n-2)...(n+m+1-i)\binom{n}{0}-(m+n-1)(m+n-2)...(n+m-1-i)\binom{n}{1}+$$

$$+(m+n-2)(m+n-3)...(n+m-2-i)\binom{n}{2}-...\pm m(m-1)(m-2)...(m-i+1)\binom{n}{n}=$$

$$=\begin{cases}0,\ ha\ i<n\\ n!,\ ha\ i=n\end{cases},\ m\geq n$$

Bizonyítás

A **Tétel 4** bizonyításánál felírt azonosságból következik abban az esetben, ha előbb a t, változók szerint deriválunk n-szer, és a t, s változók értékét egyenlővé tesszük 1-el.

Tétel 6

$$l(l-1)(l-2)...(l-i+1)\binom{n}{0}-(l+1)l(l-1)...(l-i+2)\binom{n}{1}+$$

$$+(l+2)(l+1)l...(l-i+3)\binom{n}{2}-...\pm(l+n)(l+n-1)...(n+l+1-i)\binom{n}{n}=$$

$$=\begin{cases}0,\ \ ha\ i<n\\ -n!,\ \ ha\ i=n,\ n=2k-1,\ k=1,2,3,... \\ n!,\ ha\ i=n,\ \ n=2k,\ k=1,2,3,...\end{cases}\ ;\ l\geq n$$

Bizonyítás

A **Tétel 4** bizonyításánál felírt azonosságból következik abban az esetben, ha előbb az s, változók szerint deriválunk n-szer,és a t, s változók értékét egyenlővé tesszük 1-el.

Példák

A **Tétel 1,2,3** eseteire

n=1

$$(m_1+1)\binom{1}{0}-m_1\binom{1}{1}=2\cdot1-1\cdot1=1=1!$$

$$l_1\binom{1}{0}-(l_1+1)\binom{1}{1}=1\cdot1-2\cdot1=-1=-1!$$

n=2

$$(m_1+2)\binom{2}{0}-(m_1+1)\binom{2}{1}+m_1\binom{2}{2}=3\cdot1-2\cdot2+1\cdot1=0$$

$$(m_2+2)\binom{2}{0}-(m_2+1)\binom{2}{1}+m_2\binom{2}{2}=3\cdot1-2\cdot2+1\cdot1=0$$

$$l_1\binom{2}{0}-(l_1+1)\binom{2}{1}+(l_1+2)\binom{2}{2}=1\cdot1-2\cdot2+3\cdot1=0$$

$$l_2\binom{2}{0}-(l_2+1)\binom{2}{1}+(l_2+2)\binom{2}{2}=1\cdot1-2\cdot2+3\cdot1=0$$

$$(m_1+2)(m_2+2)\binom{2}{0}-(m_1+1)(m_2+1)\binom{2}{1}+m_1m_2\binom{2}{2}=3\cdot3\cdot1-2\cdot2\cdot2+1\cdot1\cdot1=2=2!$$

$$l_1l_2\binom{2}{0}-(l_1+1)(l_2+1)\binom{2}{1}+(l_1+2)(l_2+2)\binom{2}{2}=1\cdot1\cdot1-2\cdot2\cdot2+3\cdot3\cdot1=2=2!$$

$$(m_1+2)l_1\binom{2}{0}-(m_1+1)(l_1+1)\binom{2}{1}+m_1(l_1+2)\binom{2}{2}=3\cdot1\cdot1-2\cdot2\cdot2+1\cdot3\cdot1=-2=-2!$$

$$(m_1+2)l_2\binom{2}{0}-(m_1+1)(l_2+1)\binom{2}{1}+m_1(l_2+2)\binom{2}{2}=3\cdot1\cdot1-2\cdot2\cdot2+1\cdot3\cdot1=-2=-2!$$

$$(m_2+2)l_1\binom{2}{0}-(m_2+1)(l_1+1)\binom{2}{1}+m_2(l_1+2)\binom{2}{2}=3\cdot1\cdot1-2\cdot2\cdot2+1\cdot3\cdot1=-2=-2!$$

$$(m_2+2)l_2\binom{2}{0}-(m_2+1)(l_2+1)\binom{2}{1}+m_2(l_2+2)\binom{2}{2}=3\cdot1\cdot1-2\cdot2\cdot2+1\cdot3\cdot1=-2=-2!$$

$$(m_1+2)(m_2+2)l_1\binom{2}{0}-(m_1+1)(m_2+1)(l_1+1)\binom{2}{1}+m_1m_2(l_1+2)\binom{2}{2}=$$
$$=3\cdot3\cdot1\cdot1-2\cdot2\cdot2\cdot2+1\cdot1\cdot3\cdot1=-4$$

$$(m_1+2)(m_2+2)l_2\binom{2}{0}-(m_1+1)(m_2+1)(l_2+1)\binom{2}{1}+m_1m_2(l_2+2)\binom{2}{2}=$$
$$=3\cdot3\cdot1\cdot1-2\cdot2\cdot2\cdot2+1\cdot1\cdot3\cdot1=-4$$

$$(m_1+2)l_1l_2\binom{2}{0}-(m_1+1)(l_1+1)(l_2+1)\binom{2}{1}+m_1(l_1+2)(l_2+2)\binom{2}{2}=$$
$$=3\cdot1\cdot1\cdot1-2\cdot2\cdot2\cdot2+1\cdot3\cdot3\cdot1=-2=-2!$$

$$(m_2+2)l_1l_2\binom{2}{0}-(m_2+1)(l_1+1)(l_2+1)\binom{2}{1}+m_2(l_1+2)(l_2+2)\binom{2}{2}=$$
$$=3\cdot1\cdot1-2\cdot2\cdot2+1\cdot3\cdot1=-2=-2!$$

$$(m_1+2)(m_2+2)l_1l_2\binom{2}{0}-(m_1+1)(m_2+1)(l_1+1)(l_2+1)\binom{2}{1}+m_1m_2(l_1+2)(l_2+2)\binom{2}{2}=$$
$$=3\cdot3\cdot1\cdot1\cdot1-2\cdot2\cdot2\cdot2\cdot2+1\cdot1\cdot3\cdot3\cdot1=-14$$

n=3

$$t_1^{m_1+3}t_2^{m_2+3}t_3^{m_3+3}s_1^{l_1}s_2^{l_2}s_3^{l_3}\binom{3}{0}-t_1^{m_1+2}t_2^{m_2+2}t_3^{m_3+2}s_1^{l_1+1}s_2^{l_2+1}s_3^{l_3+1}\binom{3}{1}+t_1^{m_1+1}t_2^{m_2+1}t_3^{m_3+1}s_1^{l_1+2}s_2^{l_2+2}s_3^{l_3+2}\binom{3}{2}-$$
$$-t_1^{m_1}t_2^{m_2}t_3^{m_3}s_1^{l_1+3}s_2^{l_2+3}s_3^{l_3+3}\binom{3}{3}$$

$$(m_1+3)\binom{3}{0}-(m_1+2)\binom{3}{1}+(m_1+1)\binom{3}{2}-m_1\binom{3}{3}=4\cdot1-3\cdot3+2\cdot3-1\cdot1=0$$

$$l_1\binom{3}{0}-(l_1+1)\binom{3}{1}+(l_1+2)\binom{3}{2}-(l_1+3)\binom{3}{3}=1\cdot1-2\cdot3+3\cdot3-4\cdot1=0$$

$$(m_1+3)(m_2+3)\binom{3}{0}-(m_1+2)(m_2+2)\binom{3}{1}+(m_1+1)(m_2+1)\binom{3}{2}-m_1m_2\binom{3}{3}=$$
$$=4\cdot4\cdot1-3\cdot3\cdot3+2\cdot2\cdot3-1\cdot1\cdot1=0$$

$$l_1 l_2 \binom{3}{0} - (l_1+1)(l_2+1)\binom{3}{1} + (l_1+2)(l_2+2)\binom{3}{2} - (l_1+3)(l_2+3)\binom{3}{3} = 1\cdot 1\cdot 1 - 2\cdot 2\cdot 3 + 3\cdot 3\cdot 3 - 4\cdot 4\cdot 1 = 0$$

$$(m_1+3)l_1\binom{3}{0} - (m_1+2)(l_1+1)\binom{3}{1} + (m_1+1)(l_1+2)\binom{3}{2} - m_1(l_1+3)\binom{3}{3} =$$
$$= 4\cdot 1\cdot 1 - 3\cdot 2\cdot 3 + 2\cdot 3\cdot 3 - 1\cdot 4\cdot 1 = 0$$

$$(m_1+3)(m_2+3)(m_3+3)\binom{3}{0} - (m_1+2)(m_2+2)(m_3+2)\binom{3}{1} +$$
$$+(m_1+1)(m_2+1)(m_3+1)\binom{3}{2} - m_1 m_2 m_3\binom{3}{3} = 4\cdot 4\cdot 4\cdot 1 - 3\cdot 3\cdot 3\cdot 3 + 2\cdot 2\cdot 2\cdot 3 - 1\cdot 1\cdot 1\cdot 1 = 6 = 3!$$

$$l_1 l_2 l_3\binom{3}{0} - (l_1+1)(l_2+1)(l_3+1)\binom{3}{1} + (l_1+2)(l_2+2)(l_3+2)\binom{3}{2} -$$
$$-(l_1+3)(l_2+3)(l_3+3)\binom{3}{3} = 1\cdot 1\cdot 1\cdot 1 - 2\cdot 2\cdot 2\cdot 3 + 3\cdot 3\cdot 3\cdot 3 - 4\cdot 4\cdot 4\cdot 1 = -6 = -3!$$

$$(m_1+3)(m_2+3)l_1\binom{3}{0} - (m_1+2)(m_2+2)(l_1+1)\binom{3}{1} +$$
$$+(m_1+1)(m_2+1)(l_1+2)\binom{3}{2} - m_1 m_2(l_1+3)\binom{3}{3} = 4\cdot 4\cdot 1\cdot 1 - 3\cdot 3\cdot 2\cdot 3 + 2\cdot 2\cdot 3\cdot 3 - 1\cdot 1\cdot 1\cdot 4 = -6 = -3!$$

$$(m_1+3)l_1 l_2\binom{3}{0} - (m_1+2)(l_1+1)(l_2+1)\binom{3}{1} + (m_1+1)(l_1+2)(l_2+2)\binom{3}{2} -$$
$$-m_1(l_1+3)(l_2+3)\binom{3}{3} = 4\cdot 1\cdot 1\cdot 1 - 3\cdot 2\cdot 2\cdot 3 + 2\cdot 3\cdot 3\cdot 3 - 1\cdot 4\cdot 4\cdot 1 = 6 = 3!$$

$$(m_1+3)(m_2+3)(m_3+3)l_1\binom{3}{0} - (m_1+2)(m_2+2)(m_3+2)(l_1+1)\binom{3}{1} +$$
$$+(m_1+1)(m_2+1)(m_3+1)(l_1+2)\binom{3}{2} - m_1 m_2 m_3(l_1+3)\binom{3}{3} =$$
$$= 4\cdot 4\cdot 4\cdot 1\cdot 1 - 3\cdot 3\cdot 3\cdot 2\cdot 3 + 2\cdot 2\cdot 2\cdot 3\cdot 3 - 1\cdot 1\cdot 1\cdot 1\cdot 4 = -30$$

$$(m_1+3)l_1 l_2 l_3\binom{3}{0} - (m_1+2)(l_1+1)(l_2+1)(l_3+1)\binom{3}{1} + (m_1+1)(l_1+2)(l_2+2)(l_3+2)\binom{3}{2} -$$
$$-m_1(l_1+3)(l_2+3)(l_3+3)\binom{3}{3} = 4\cdot 1\cdot 1\cdot 1\cdot 1 - 3\cdot 2\cdot 2\cdot 2\cdot 3 + 2\cdot 3\cdot 3\cdot 3\cdot 3 - 1\cdot 4\cdot 4\cdot 4\cdot 1 = 30$$

$$(m_1+3)(m_2+3)l_1 l_2\binom{3}{0} - (m_1+2)(m_2+2)(l_1+1)(l_2+1)\binom{3}{1} + (m_1+1)(m_2+1)(l_1+2)(l_2+2)\binom{3}{2} -$$
$$-m_1 m_2(l_1+3)(l_2+3)\binom{3}{3} = 4\cdot 4\cdot 1\cdot 1\cdot 1 - 3\cdot 3\cdot 2\cdot 2\cdot 3 + 2\cdot 2\cdot 3\cdot 3\cdot 3 - 1\cdot 1\cdot 4\cdot 4\cdot 1 = 0$$

$$(m_1+3)(m_2+3)(m_3+3)l_1 l_2\binom{3}{0} - (m_1+2)(m_2+2)(m_3+2)(l_1+1)(l_2+1)\binom{3}{1} +$$
$$+(m_1+1)(m_2+1)(m_3+1)(l_1+2)(l_2+2)\binom{3}{2} - m_1 m_2 m_3(l_1+3)(l_2+3)\binom{3}{3} =$$
$$= 4\cdot 4\cdot 4\cdot 1\cdot 1\cdot 1 - 3\cdot 3\cdot 3\cdot 2\cdot 2\cdot 3 + 2\cdot 2\cdot 2\cdot 3\cdot 3\cdot 3 - 1\cdot 1\cdot 1\cdot 4\cdot 4\cdot 1 = -60$$

$$(m_1+3)(m_2+3)l_1l_2l_3\binom{3}{0}-(m_1+2)(m_2+2)(l_1+1)(l_2+1)(l_3+1)\binom{3}{1}+$$

$$+(m_1+1)(m_2+1)(l_1+2)(l_2+2)(l_3+2)\binom{3}{2}-m_1m_2(l_1+3)(l_2+3)(l_3+3)\binom{3}{3}=$$

$$=4\cdot4\cdot1\cdot1\cdot1\cdot1\cdot1-3\cdot3\cdot2\cdot2\cdot2\cdot3+2\cdot2\cdot3\cdot3\cdot3\cdot3-1\cdot1\cdot4\cdot4\cdot4\cdot1=60$$

$$(m_1+3)(m_2+3)(m_3+3)l_1l_2l_3\binom{3}{0}-(m_1+2)(m_2+2)(m_3+2)(l_1+1)(l_2+1)(l_3+1)\binom{3}{1}+$$

$$+(m_1+1)(m_2+1)(m_3+1)(l_1+2)(l_2+2)(l_3+2)\binom{3}{2}-m_1m_2m_3(l_1+3)(l_2+3)(l_3+3)\binom{3}{3}=$$

$$=4\cdot4\cdot4\cdot1\cdot1\cdot1\cdot1\cdot1-3\cdot3\cdot3\cdot2\cdot2\cdot2\cdot3+2\cdot2\cdot2\cdot3\cdot3\cdot3\cdot3-1\cdot1\cdot1\cdot4\cdot4\cdot4\cdot1=0$$

n=4

$$(m_1+4)(m_2+4)(m_3+4)(m_4+4)l_1l_2l_3l_4\binom{4}{0}-$$

$$-(m_1+3)(m_2+3)(m_3+3)(m_4+3)(l_1+1)(l_2+1)(l_3+1)(l_4+1)\binom{4}{1}+$$

$$+(m_1+2)(m_2+2)(m_3+2)(m_4+2)(l_1+2)(l_2+2)(l_3+2)(l_4+2)\binom{4}{2}-$$

$$-(m_1+1)(m_2+1)(m_3+1)(m_4+1)(l_1+3)(l_2+3)(l_3+3)(l_4+3)\binom{4}{3}+$$

$$+m_1m_2m_3m_4(l_1+3)(l_2+3)(l_3+3)(l_4+3)\binom{4}{4}=$$

$$=5\cdot5\cdot5\cdot5\cdot1\cdot1\cdot1\cdot1\cdot1-4\cdot4\cdot4\cdot4\cdot2\cdot2\cdot2\cdot2\cdot4+3\cdot3\cdot3\cdot3\cdot3\cdot3\cdot3\cdot3\cdot6-$$

$$-2\cdot2\cdot2\cdot2\cdot4\cdot4\cdot4\cdot4\cdot4+1\cdot1\cdot1\cdot1\cdot4\cdot4\cdot4\cdot4\cdot1=7469$$

Példák

A **Tétel 4,5,6** eseteire

n=1

$$(m+1)\binom{1}{0}-m\binom{1}{1}=2\cdot1-1\cdot1=1$$

$$l\binom{1}{0}-(l+1)\binom{1}{1}=1\cdot1-2\cdot1=-1$$

$$(m+1)l\binom{1}{0}-m(l+1)\binom{1}{1}=2\cdot1\cdot1-1\cdot2\cdot1=0$$

n=2

$$t^ms^l(t-s)^2=t^{m+2}s^l\binom{2}{0}-t^{m+1}s^{l+1}\binom{2}{1}+t^ms^{l+2}\binom{2}{2}$$

$$(m+2)\binom{2}{0}-(m+1)\binom{2}{1}+m\binom{2}{2}=4\cdot1-3\cdot2+2\cdot1=0$$

$$l\binom{2}{0}-(l+1)\binom{2}{1}+(l+2)\binom{2}{2}=2\cdot 1-3\cdot 2+4\cdot 1=0$$

$$(m+2)(m+1)\binom{2}{0}-(m+1)m\binom{2}{1}+m(m-1)\binom{2}{2}=4\cdot 3\cdot 1-3\cdot 2\cdot 2+2\cdot 1\cdot 1=2=2!$$

$$l(l-1)\binom{2}{0}-(l+1)l\binom{2}{1}+(l+2)(l+1)\binom{2}{2}=2\cdot 1\cdot 1-3\cdot 2\cdot 2+4\cdot 3\cdot 1=2=2!$$

$$(m+2)l\binom{2}{0}-(m+1)(l+1)\binom{2}{1}+m(l+2)\binom{2}{2}=4\cdot 2\cdot 1-3\cdot 3\cdot 2+2\cdot 4\cdot 1=-2=-2!$$

n=3

$$t^m s^l (t-s)^3=t^{m+3}s^l\binom{3}{0}-t^{m+2}s^{l+1}\binom{3}{1}+t^{m+1}s^{l+2}\binom{3}{2}-t^m s^{l+3}\binom{3}{3}$$

$$(m+3)\binom{3}{0}-(m+2)\binom{3}{1}+(m+1)\binom{3}{2}-m\binom{3}{3}=6\cdot 1-5\cdot 3+4\cdot 3-3\cdot 1=0$$

$$l\binom{3}{0}-(l+1)\binom{3}{1}+(l+2)\binom{3}{2}-(l+3)\binom{3}{3}=3\cdot 1-4\cdot 3+5\cdot 3-6\cdot 1=0$$

$$(m+3)(m+2)\binom{3}{0}-(m+2)(m+1)\binom{3}{1}+(m+1)m\binom{3}{2}-m(m-1)\binom{3}{3}=6\cdot 5\cdot 1-5\cdot 4\cdot 3+4\cdot 3\cdot 3-3\cdot 2\cdot 1=0$$

$$l(l-1)\binom{3}{0}-(l+1)l\binom{3}{1}+(l+2)(l+1)\binom{3}{2}-(l+3)(l+2)\binom{3}{3}=3\cdot 2\cdot 1-4\cdot 3\cdot 3+5\cdot 4\cdot 3-6\cdot 5\cdot 1=0$$

$$(m+3)l\binom{3}{0}-(m+2)(l+1)\binom{3}{1}+(m+1)(l+2)\binom{3}{2}-m(l+3)\binom{3}{3}=6\cdot 3\cdot 1-5\cdot 4\cdot 3+4\cdot 5\cdot 3-3\cdot 6\cdot 1=0$$

$$(m+3)(m+2)(m+1)\binom{3}{0}-(m+2)(m+1)m\binom{3}{1}+(m+1)m(m-1)\binom{3}{2}-m(m-1)(m-2)\binom{3}{3}=$$

$$=6\cdot 5\cdot 4\cdot 1-5\cdot 4\cdot 3\cdot 3+4\cdot 3\cdot 2\cdot 3-3\cdot 2\cdot 1\cdot 1=6=3!$$

$$l(l-1)(l-2)\binom{3}{0}-(l+1)l(l-1)\binom{3}{1}+(l+2)(l+1)l\binom{3}{2}-(l+3)(l+2)(l+1)\binom{3}{3}=$$

$$=3\cdot 2\cdot 1\cdot 1-4\cdot 3\cdot 2\cdot 3+5\cdot 4\cdot 3\cdot 3-6\cdot 5\cdot 4\cdot 1=-6=-3!$$

$$(m+3)(m+2)l\binom{3}{0}-(m+2)(m+1)(l+1)\binom{3}{1}+(m+1)m(l+2)\binom{3}{2}-m(m-1)(l+3)\binom{3}{3}=$$

$$=6\cdot 5\cdot 3\cdot 1-5\cdot 4\cdot 4\cdot 3+4\cdot 3\cdot 5\cdot 3-3\cdot 2\cdot 6\cdot 1=90-240+180-36=270-276=-6=-3!$$

$$(m+3)l(l-1)\binom{3}{0}-(m+2)(l+1)l\binom{3}{1}+(m+1)(l+2)(l+1)\binom{3}{2}-m(l+3)(l+2)\binom{3}{3}=$$

$$=6\cdot 3\cdot 2\cdot 1-5\cdot 4\cdot 3\cdot 3+4\cdot 5\cdot 4\cdot 3-3\cdot 6\cdot 5\cdot 1=36-180+240-90=276-270=6=3!$$

Megjegyzések

A magasabb fokú, többdimenziós logisztikus egyenletek felírásával, illetve azok átírásával a binomiális tétel alapján több állítást is felírhatunk.

Az **Tétel 1** alapján

$m_1=m_2=m_3=\dots m_n=m$

$$(m+n)^i l_1 l_2 l_3 \ldots l_j \binom{n}{0} - (m+n-1)^i (l_1+1)(l_2+1)(l_3+1)\ldots(l_j+1)\binom{n}{1} +$$

$$+(m+n-2)^i (l_1+2)(l_2+2)(l_3+2)\ldots(l_j+2)\binom{n}{2} - \ldots \pm m^i (l_1+n)(l_2+n)(l_3+n)\ldots(l_j+n)\binom{n}{n} =$$

$$=\begin{cases} 0,\ ha\ i+j<n \\ -n!,\ ha\ i+j=n,\ j=2k-1,\ k=1,2,3,\ldots \\ n!,\ ha\ i+j=n,\ j=2k,\ k=1,2,3,\ldots \\ <0,\ ha\ n<i+j<2n,\ n=2k,\ k=1,2,3,\ldots \\ <0,\ ha\ n<i+j<2n,\ n=2k-1,\ k=1,2,3,\ldots \\ >0,\ ha\ n<i+j<2n,\ n=2k-1,\ j=2k-1,\ k=1,2,3,\ldots \\ 0,\ ha\ i+j=2n,\ n=1,3,5,\ldots \\ <0,\ ha\ i+j=2n,\ n=2k-1,\ k=1,2,3,\ldots \\ >0,\ ha\ i+j=2n,\ n=2k,\ k=1,2,3,\ldots \end{cases}$$

$l_1=l_2=l_3=\ldots l_n=l$

$$(m_1+n)(m_2+n)(m_3+n)\ldots(m_i+n)l^j\binom{n}{0} - (m_1+n-1)(m_2+n-1)(m_3+n-1)\ldots(m_i+n-1)(l+1)^j\binom{n}{1} +$$

$$+(m_1+n-2)(m_2+n-2)(m_3+n-2)\ldots(m_i+n-2)(l+2)^j\binom{n}{2} - \ldots \pm m_1 m_2 m_3 \ldots m_i (l+n)^j\binom{n}{n} =$$

$$=\begin{cases} 0,\ ha\ i+j<n \\ -n!,\ ha\ i+j=n,\ j=2k-1,\ k=1,2,3,\ldots \\ n!,\ ha\ i+j=n,\ j=2k,\ k=1,2,3,\ldots \\ <0,\ ha\ n<i+j<2n,\ n=2k,\ k=1,2,3,\ldots \\ <0,\ ha\ n<i+j<2n,\ n=2k-1,\ k=1,2,3,\ldots \\ >0,\ ha\ n<i+j<2n,\ n=2k-1,\ j=2k-1,\ k=1,2,3,\ldots \\ 0,\ ha\ i+j=2n,\ n=1,3,5,\ldots \\ <0,\ ha\ i+j=2n,\ n=2k-1,\ k=1,2,3,\ldots \\ >0,\ ha\ i+j=2n,\ n=2k,\ k=1,2,3,\ldots \end{cases}$$

$m_1=m_2=m_3=\ldots m_n=m$, $l_1=l_2=l_3=\ldots l_n=l$

$$(m+n)^i l^j\binom{n}{0} - (m+n-1)^i (l+1)^j\binom{n}{1} + (m+n-2)^i (l+2)^j\binom{n}{2} - \ldots \pm m^i (l+n)^j\binom{n}{n} =$$

$$=\begin{cases} 0,\ ha\ i+j<n \\ -n!,\ ha\ i+j=n,\ j=2k-1,\ k=1,2,3,\ldots \\ n!,\ ha\ i+j=n,\ j=2k,\ k=1,2,3,\ldots \\ <0,\ ha\ n<i+j<2n,\ n=2k,\ k=1,2,3,\ldots \\ <0,\ ha\ n<i+j<2n,\ n=2k-1,\ k=1,2,3,\ldots \\ >0,\ ha\ n<i+j<2n,\ n=2k-1,\ j=2k-1,\ k=1,2,3,\ldots \\ 0,\ ha\ i+j=2n,\ n=1,3,5,\ldots \\ <0,\ ha\ i+j=2n,\ n=2k-1,\ k=1,2,3,\ldots \\ >0,\ ha\ i+j=2n,\ n=2k,\ k=1,2,3,\ldots \end{cases}$$

$t_1=t_2=t_3=\ldots=t_n=t$

$$(m_1+m_2+m_3+\ldots+m_i+nn)l_1l_2l_3\ldots l_j\binom{n}{0}-$$

$$-(m_1+m_2+m_3+\ldots+m_i+n[n-1])(l_1+1)(l_2+1)(l_3+1)\ldots(l_j+1)\binom{n}{1}+$$

$$+(m_1+m_2+m_3+\ldots+m_i+n[n-2])(l_1+2)(l_2+2)(l_3+2)\ldots(l_j+2)\binom{n}{2}-$$

$$-\ldots\pm(m_1+m_2+m_3+\ldots+m_i+n[n-n])(l_1+n)(l_2+n)(l_3+n)\ldots(l_j+n)\binom{n}{n}=$$

$$=\begin{cases} 0,\ ha\ i+j<n \\ -n!,\ ha\ i+j=n,\ j=2k-1,\ k=1,2,3,\ldots \\ n!,\ ha\ i+j=n,\ j=2k,\ k=1,2,3,\ldots \\ <0,\ ha\ n<i+j<2n,\ n=2k,\ k=1,2,3,\ldots \\ <0,\ ha\ n<i+j<2n,\ n=2k-1,\ k=1,2,3,\ldots \\ >0,\ ha\ n<i+j<2n,\ n=2k-1,\ j=2k-1,\ k=1,2,3,\ldots \\ 0,\ ha\ i+j=2n,\ n=1,3,5,\ldots \\ <0,\ ha\ i+j=2n,\ n=2k-1,\ k=1,2,3,\ldots \\ >0,\ ha\ i+j=2n,\ n=2k,\ k=1,2,3,.. \end{cases}$$

A **Tétel 1** bizonyításánál felírt azonosságban a $t_1=t_2=t_3=\ldots=t_n=t$ írva, és t szerint n-szer differenciálva adódik a fenti állítás.

$s_1=s_2=s_3=\ldots=s_n=s$

$$(m_1+n)(m_2+n)(m_3+n)\ldots(m_i+n)(l_1+l_2+l_3+\ldots+l_j)\binom{n}{0}-$$

$$-(m_1+n-1)(m_2+n-1)(m_3+n-1)\ldots(m_i+n-1)(l_1+l_2+l_3+\ldots+l_j+1)\binom{n}{1}+$$

$$+(m_1+n-2)(m_2+n-2)(m_3+n-2)\ldots(m_i+n-2)(l_1+l_2+l_3+\ldots+l_j+2)\binom{n}{2}-$$

$$-\ldots\pm m_1m_2m_3\ldots m_i(l_1+l_2+l_3+\ldots+l_j+nn)\binom{n}{n}=$$

$$=\begin{cases} 0,\ ha\ i+j<n \\ -n!,\ ha\ i+j=n,\ j=2k-1,\ k=1,2,3,\ldots \\ n!,\ ha\ i+j=n,\ j=2k,\ k=1,2,3,\ldots \\ <0,\ ha\ n<i+j<2n,\ n=2k,\ k=1,2,3,\ldots \\ <0,\ ha\ n<i+j<2n,\ n=2k-1,\ k=1,2,3,\ldots \\ >0,\ ha\ n<i+j<2n,\ n=2k-1,\ j=2k-1,\ k=1,2,3,\ldots \\ 0,\ ha\ i+j=2n,\ n=1,3,5,\ldots \\ <0,\ ha\ i+j=2n,\ n=2k-1,\ k=1,2,3,\ldots \\ >0,\ ha\ i+j=2n,\ n=2k,\ k=1,2,3,\ldots \end{cases}$$

A **Tétel 1** bizonyításánál felírt azonosságban a $s_1=s_2=s_3=\ldots=s_n=s$ írva, és s szerint n-szer differenciálva adódik a fenti állítás.

$t_1=t_2=t_3=\ldots=t_n=t$, $s_1=s_2=s_3=\ldots=s_n=s$

$$(m_1+m_2+m_3+\ldots+m_i+nn)(l_1+l_2+l_3+\ldots+l_j)\binom{n}{0}-$$

$$-(m_1+m_2+m_3+\ldots+m_i+n[n-1])(l_1+l_2+l_3+\ldots+l_j+1)\binom{n}{1}+$$

$$+(m_1+m_2+m_3+\ldots+m_i+n[n-2])(l_1+l_2+l_3+\ldots+l_j+2)\binom{n}{2}-$$

$$-\ldots\pm(m_1+m_2+m_3+\ldots+m_i+n[n-n])(l_1+l_2+l_3+\ldots+l_j+nn)\binom{n}{n}=$$

$$=\begin{cases} 0, \ ha\ i+j<n \\ -n!, \ ha\ i+j=n,\ j=2k-1,\ k=1,2,3,\ldots \\ n!, \ ha\ i+j=n,\ j=2k,\ k=1,2,3,\ldots \\ <0, \ ha\ n<i+j<2n,\ n=2k,\ k=1,2,3,\ldots \\ <0, \ ha\ n<i+j<2n,\ n=2k-1,\ k=1,2,3,\ldots \\ >0, \ ha\ n<i+j<2n,\ n=2k-1,\ j=2k-1,\ k=1,2,3,\ldots \\ 0, \ ha\ i+j=2n,\ n=1,3,5,\ldots \\ <0, \ ha\ i+j=2n,\ n=2k-1,\ k=1,2,3,\ldots \\ >0, \ ha\ i+j=2n,\ n=2k,\ k=1,2,3,\ldots \end{cases}$$

A **Tétel 1** bizonyításánál felírt azonosságban a $t_1=t_2=t_3=\ldots=t_n=t$, $s_1=s_2=s_3=\ldots=s_n=s$ írva, és t szerint n-szer, s szerint is n-szer differenciálva adódik a fenti állítás.